Combined Stresses
الاجهادات المركبة

نسألكم الدعاء

Table of Contents

* Combined Stresses ---------- Page 2
* Sign Rule ---------- Page 4
* Stresses at any plane ---------- Page 5
* Principle Normal stresses and their planes--- Page 6
* Principle Shear stresses and their planes --- Page 7
* Mohr's Circle ---------- Page 8
* Examples ---------- Page 9
* Proof of Principle stresses and their planes -- Page 37

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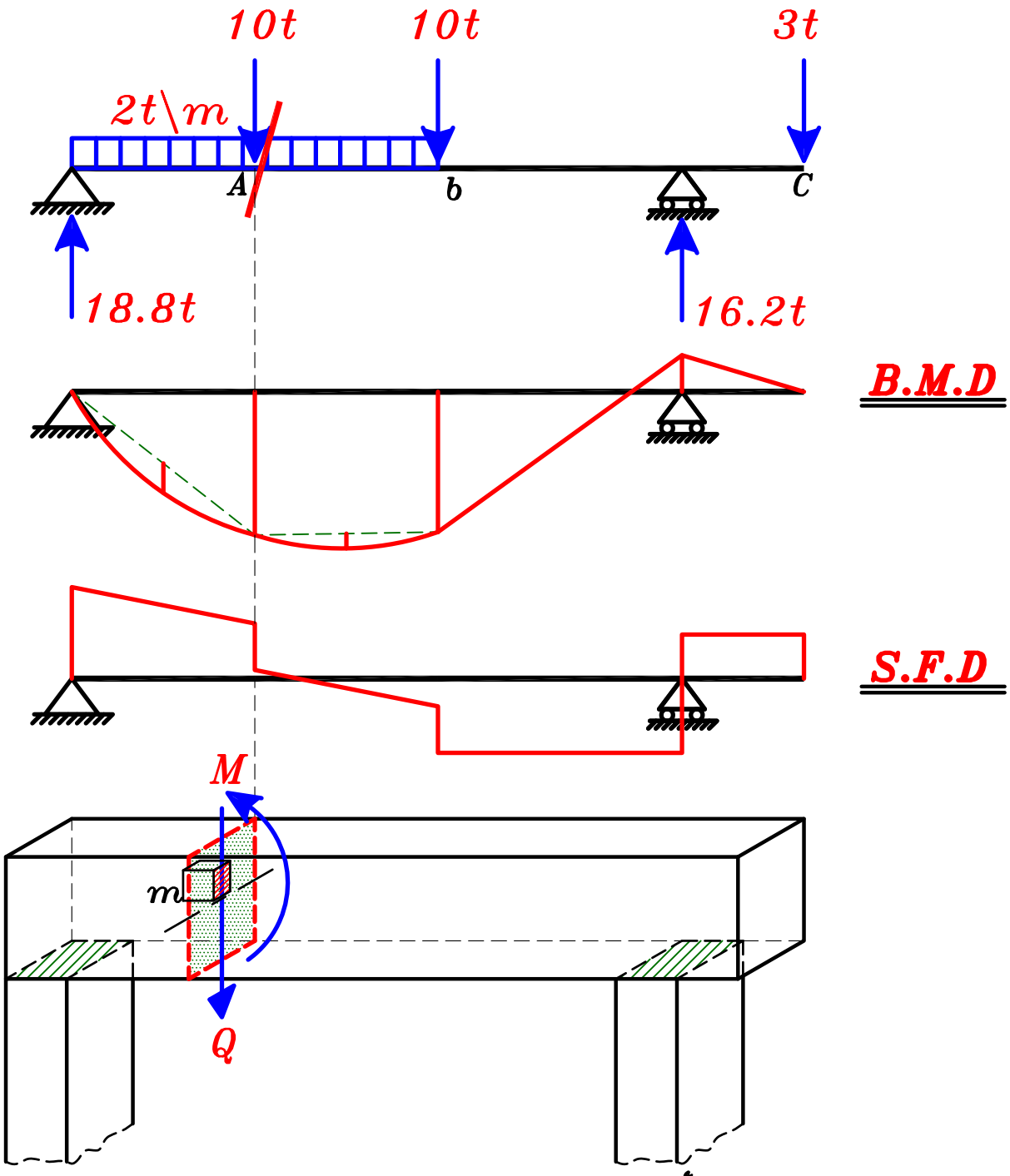
COMBINED STRESSES

أى نوع من المنشآت يكون معرض لاكثر من نوع من الـ Stresses أى يكون معرض لـ Normal Stresses و Shear Stresses أى أنه يكون معرض لتأثير ما نسميه بالـ Combined Stresses.



و بدراسة Section A نجد أنه معرض لـ Bending moment مما يسبب وجود Normal stresses به كما أنه معرض لوجود Shear Force مما يسبب وجود Shear stresses به.

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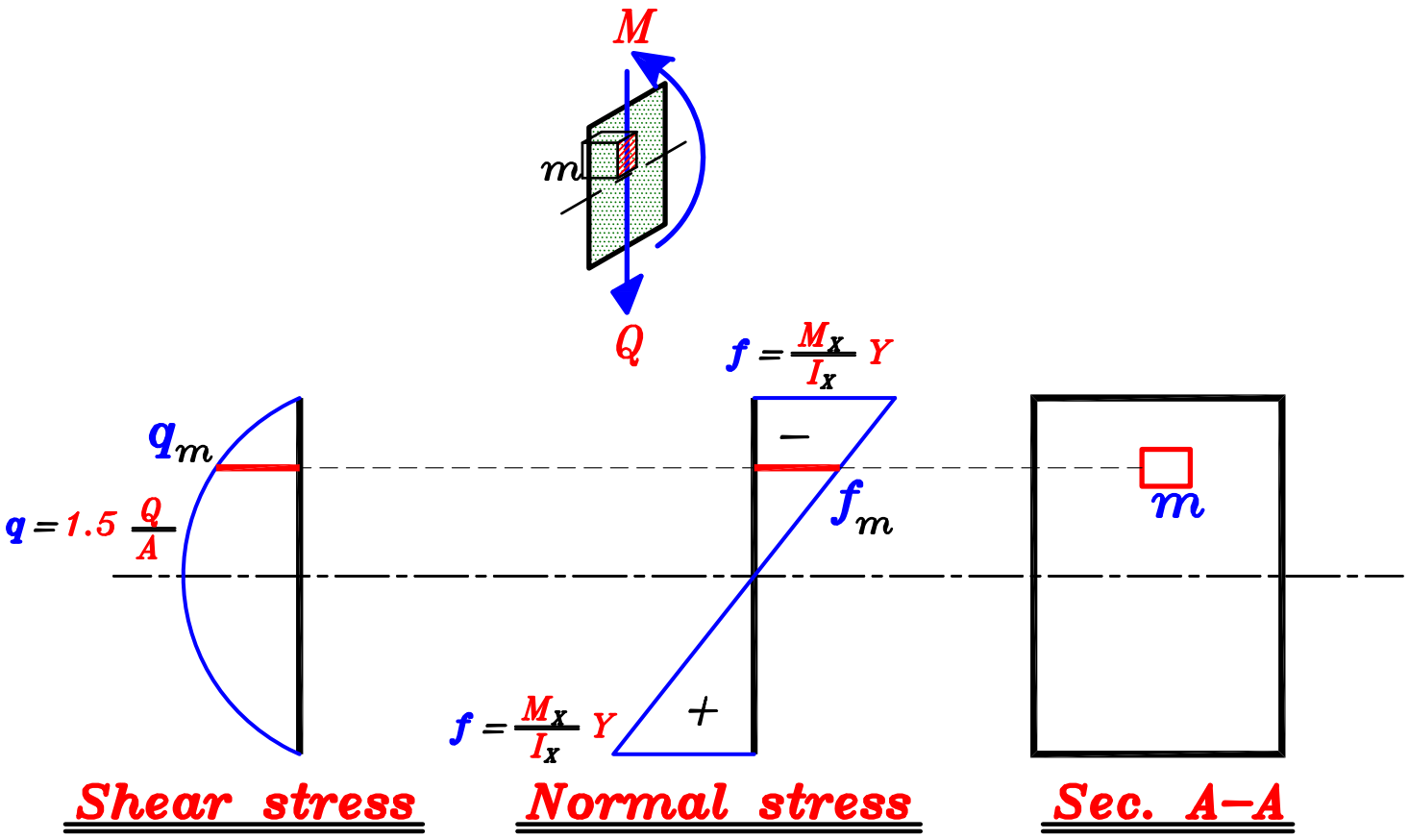
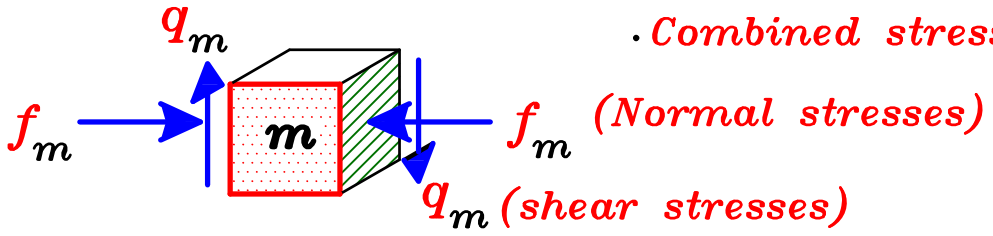
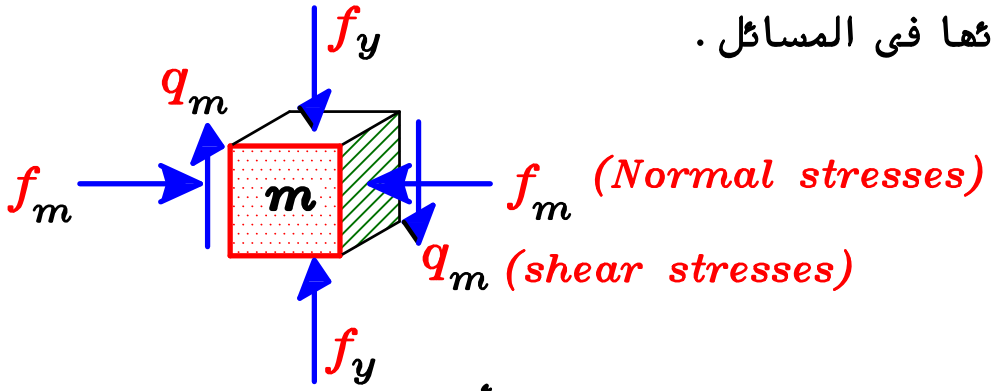


و اذا قمنا بدراسة المكعب m و هو عبارة عن نقطة صغيرة نجد أن هذا المكعب معرض لـ (f_m) Normal stresses و (q_m) Shear stresses أى أنه معرض لـ Combined stresses .



و من الممكن أن يتعرض هذا المكعب لـ Normal stresses رأسية نتيجة اتصاله بالمكعب اللى فوقه و اللى تحته و لكن دائما نهمل هذه الـ Stresses الا اذا تم اعطائها فى المسائل .



و نتيجة الـ Shear stresses الرأسية تتولد Shear stresses أفقية حتى يحدث اتزان للمكعب وهذا مادرسناه سابقا ويسمى بالـ Complementary shear و تكون مساوية للـ Shear stresses الرأسية .

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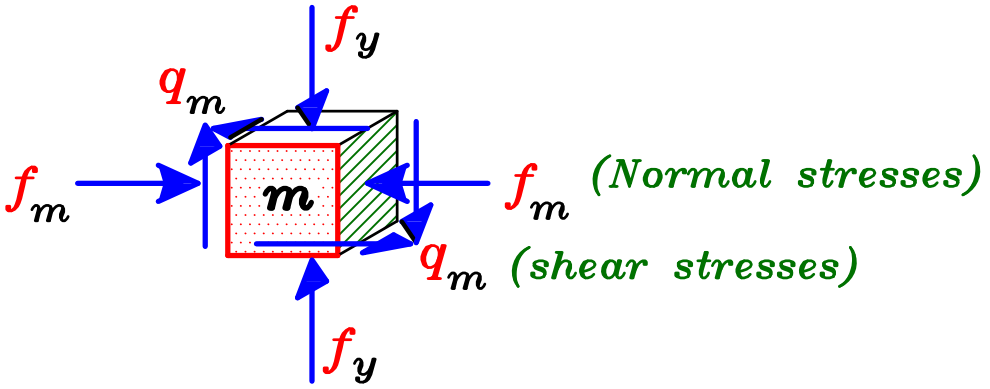
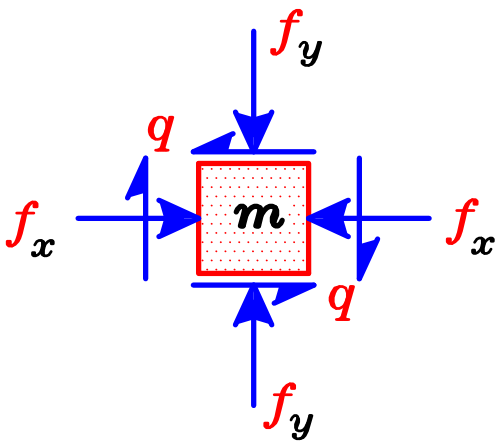


و دائما ما نسقط الوجه المنقط و نتعامل معه كالتالى



ملحوظة هامة جدا

الوجه الذى ندرسه من المكعب لابد أن يكون الـ *normal stress* عمودى على احد اضلاعه و الـ *Shear stress* موازى لاحد اضلاعه و لابد أن تظهر الـ *Stresses* على هذا الوجه خطوط وليست نقط

SIGN RULE

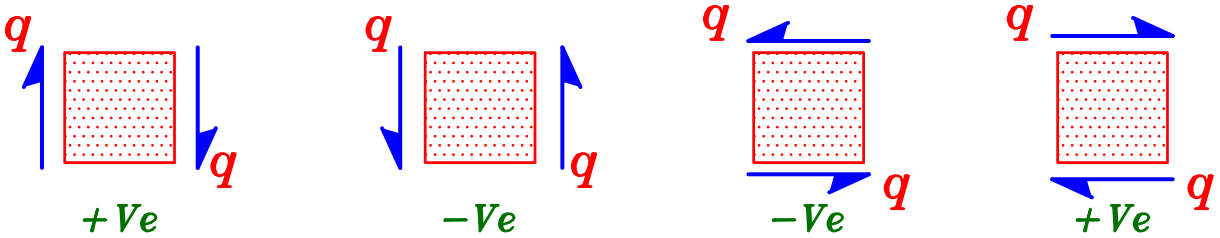
1-(Normal stresses)

f_x & f_y $\Longrightarrow$ $(-Ve) \Longrightarrow$ *Compression*

$(+Ve) \Longrightarrow$ *Tension*

2-(Shear stresses)

فى حالة اذا كان اتجاه دوران الـ *Couple* فى اتجاه عقارب الساعة تكون اشارة الـ *Shear stresses* بالـ $+Ve$ و ذا كان عكس عقارب الساعة تكون اشارة الـ *Shear stresses* بالـ $-Ve$.



و دائما فى المسائل نأخذ اشارة الـ *Shear stresses* الرأسية لانها الاصل و هى التى يتولد منها الـ *Shear stresses* الافقية .

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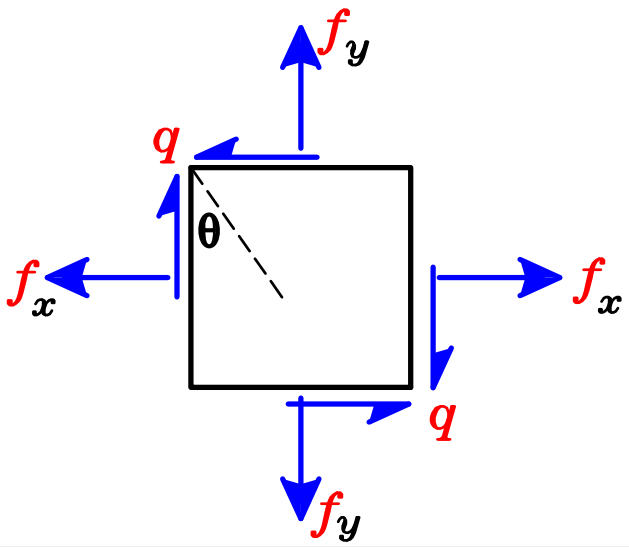
Stresses at any plane

و فى هذا الدرس نتعلم كيفية حساب الـ Normal stresses و الـ Shear stresses على أى مستوى مائل كما نتعلم كيفية حساب الـ max. Normal stresses و المستوى الذى تؤثر عليه و أيضا الـ max. Shear stresses و المستوى الذى تؤثر عليه و نحسب كل هذا من مجموعة من القوانين و اثباتها موجود فى نهاية الملزمة.

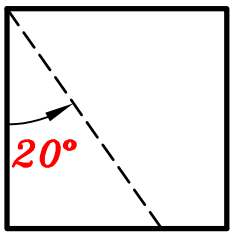


حفظ

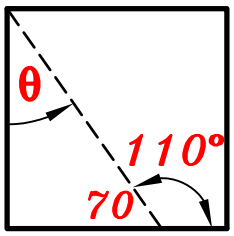
$$f_\theta = \left[\frac{f_x + f_y}{2}\right] + \left[\frac{f_x - f_y}{2}\right] \cos 2\theta - q \sin 2\theta$$

$$q_\theta = \left[\frac{f_x - f_y}{2}\right] \sin 2\theta + q \cos 2\theta$$

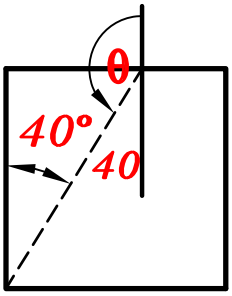
* و الزاوية مقاسة مع المحور الرأسى عكس عقارب الساعة



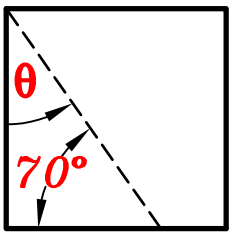
Anti clockwise
θ = 20



Anti clockwise
θ = 20



Anti clockwise
θ = 140



Anti clockwise
θ = 20

* و بتفاضل معادلة الـ f_θ و مساوتها بالصفر يمكن ايجاد الزاوية التى تعطى مستوى $f_{min.}$ & $f_{max.}$ و بالتعويض بقيمة هذه الزاوية فى معادلة الـ f_θ نوجد قيمة الـ $f_{min.}$ & $f_{max.}$ و بنفس الطريقة يمكن ايجاد المستوى الذى عليه $q_{min.}$ & $q_{max.}$ و كذلك حساب $q_{min.}$ & $q_{max.}$.

و هذا موجود فى الاثبات قى نهاية الملزمة .

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Principle Normal stresses and their planes

$$f_{max.} = \left[\frac{f_x + f_y}{2} \right] + \sqrt{\left[\frac{f_x - f_y}{2} \right]^2 + q^2}$$

$$f_{min.} = \left[\frac{f_x + f_y}{2} \right] - \sqrt{\left[\frac{f_x - f_y}{2} \right]^2 + q^2}$$

$$\tan 2\theta = \frac{-2q}{f_x - f_y}$$

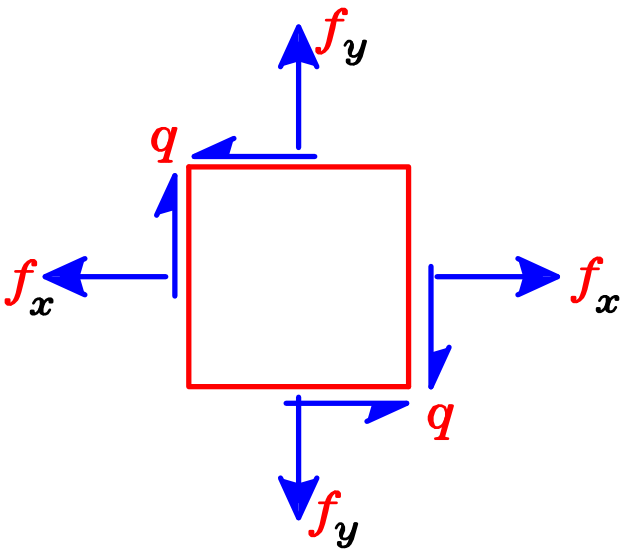
حفظ

* و الزاوية θ هى الزاوية بين المستوى الافقى أو الرأسى و المستوى الذى عليه الـ $f_{max.}$ or $f_{min.}$ و مقاسة عكس عقارب الساعة .

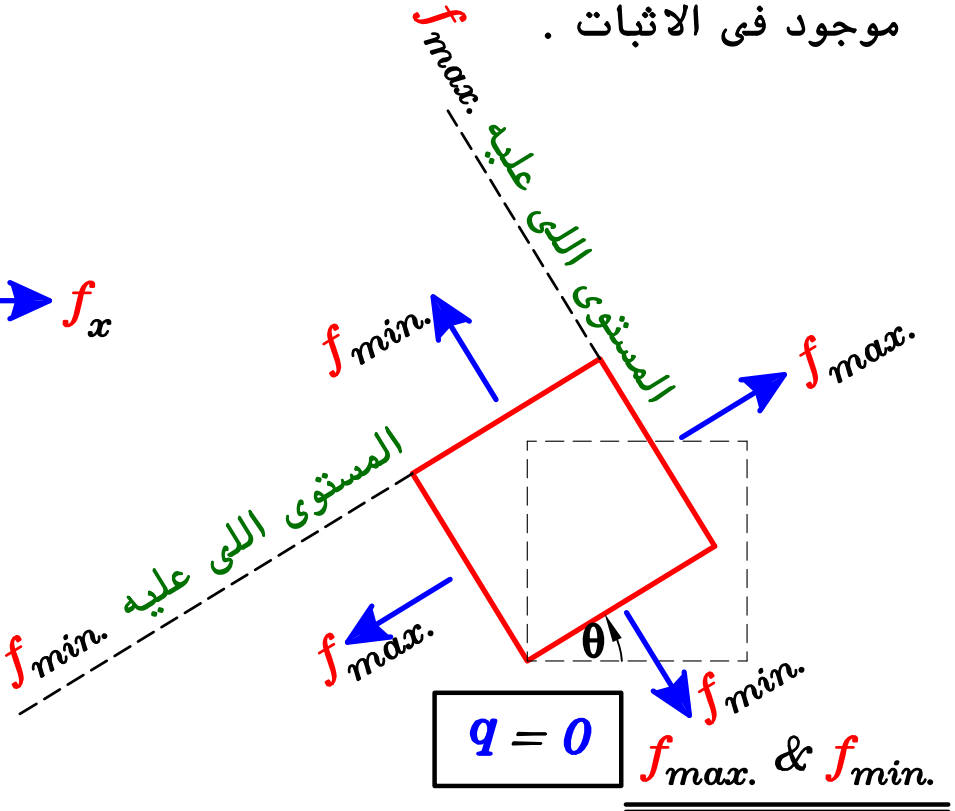
* للوصول للمستوى الذى عليه $f_{max.}$ تقاس الـ θ من المستوى اللى عليه الكبير من الـ f_x or f_y .

* المستوى الذى عليه $f_{min.}$ يكون عمودى على المستوى اللى عليه الـ $f_{max.}$ أو نقيس الزاوية θ من المستوى اللى عليه الاصغر من الـ f_x or f_y .

* المستوى الذى عليه $f_{max.}$ or $f_{min.}$ لا يكون عليه *Shear stresses* و هذا موجود فى الاثبات .



مثلا
$f_x > f_y$
$\theta = +Ve$



$q = 0$

$f_{max.}$ & $f_{min.}$

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Principle Shear stresses and their planes

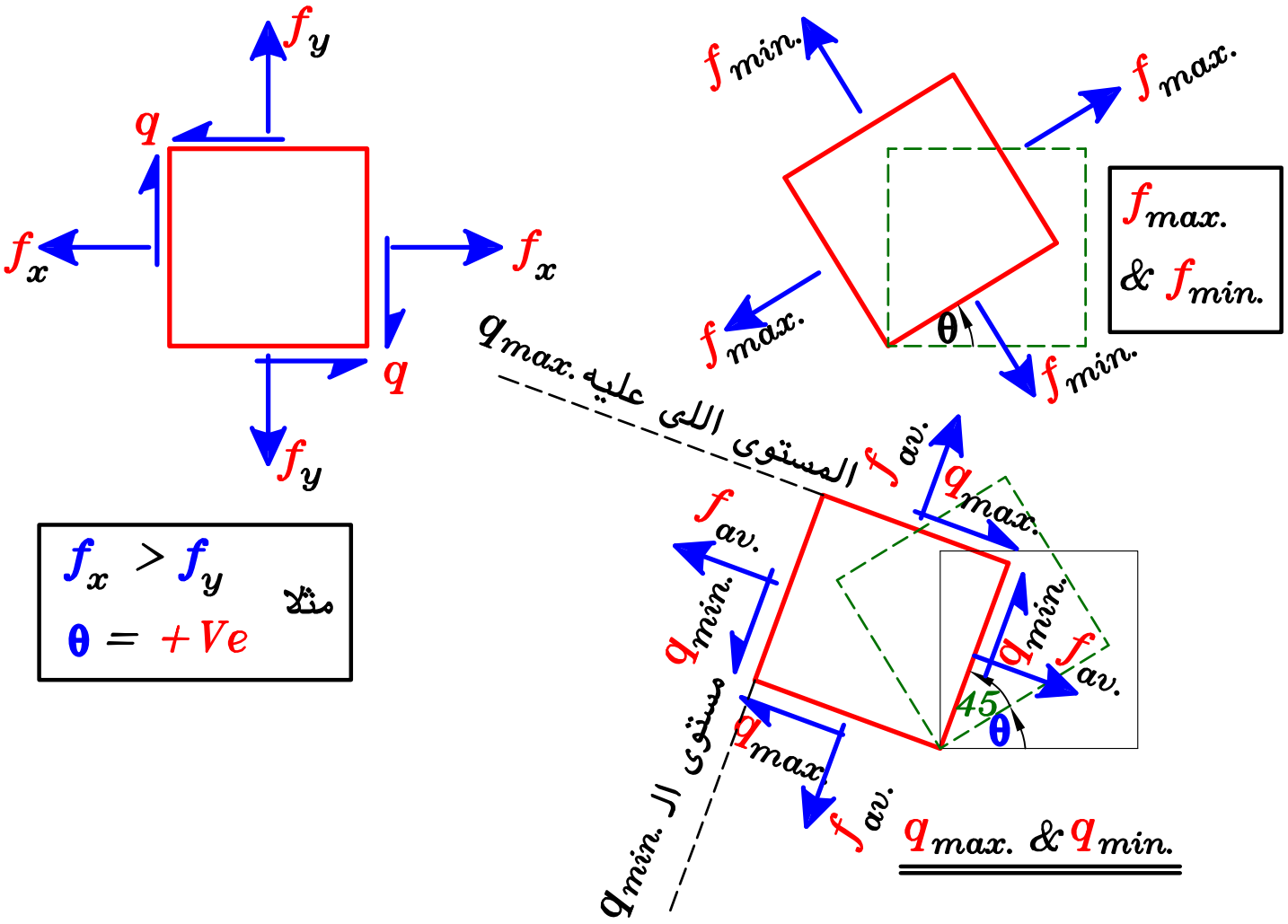
$$q_{max.} = + \sqrt{[\frac{f_x - f_y}{2}]^2 + q^2}$$

$$q_{min.} = - \sqrt{[\frac{f_x - f_y}{2}]^2 + q^2}$$

حفظ

* للوصول للمستوى الذى عليه $q_{max.}$ نقيس زاوية 45 عكس عقارب الساعة مع المستوى اللى عليه الـ $f_{max.}$.
* المستوى الذى عليه $q_{min.}$ يكون عمودى على المستوى اللى عليه الـ $q_{max.}$ أو نقيس زاوية 45 عكس عقارب الساعة مع المستوى اللى عليه الـ $f_{min.}$.
* المستوى الذى عليه $q_{max.}$ or $q_{min.}$ يكون عليه Shear stresses تساوى $f_{av.}$

$$f_{av.} = \frac{f_x + f_y}{2}$$



$f_x > f_y$ مثلا

$\theta = +Ve$

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* و من الممكن تحديد الـ *max.&min. stresses* و مستوياتها عن طريق دائرة *mohr*.

mohr's circle for stresses

١- يتم رسم محاور رئيسية حيث أن المحور الافقى يعبر عن *(f)* و المحور الافقى يعبر عن *(q)*.

٢- يتم تحديد مركز الدائرة (دائرة مور) عن طريق نقطة تبعد مسافة $\frac{f_x + f_y}{2}$ عن المحور الرأسى و باشارتها.

٣- نحدد نقطة الـ *Pole* و التى احداثيتها $(f_x , -q)$.

٤- نرسم دائرة مور حيث يكون الـ *radius* من مركز الدائرة الى الـ *Pole* و تقطع الدائرة المحور الافقى فى نقطتين احداثياتهما $(f_{max.} , 0)$ و $(f_{min.} , 0)$ كما تقطع الخط الرأسى المار بالمركز فى نقطتين احداثياتهما $(f_{av.} , q_{max.})$ و $(f_{av.} , q_{min.})$.

٥- اتجاه المستوى اللى عليه الـ *maximum normal stresses* هو الخط الواصل من الـ *Pole* و نقطة $(f_{max.} , 0)$.

٦- اتجاه المستوى اللى عليه الـ *minimum normal stresses* هو الخط الواصل من الـ *Pole* و نقطة $(f_{min.} , 0)$.

٧- اتجاه المستوى اللى عليه الـ *maximum shear stresses* هو الخط الواصل من الـ *Pole* و نقطة $(f_{av.} , q_{max.})$.

٨- اتجاه المستوى اللى عليه الـ *minimum shear stresses* هو الخط الواصل من الـ *Pole* و نقطة $(f_{av.} , q_{min.})$.

لابد من رسم الدائرة بـ *Scale*

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Example:

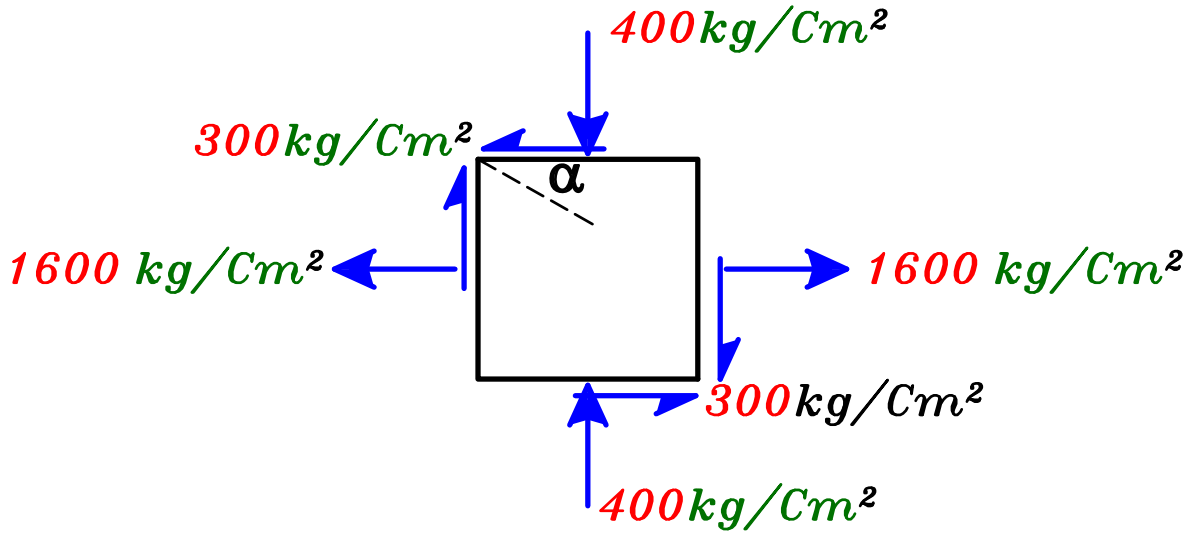
For the shown stress state it is required to find :

a) The principle normal stresses and their planes .

b) The principle shear stresses and their planes and the associated normal stresses.

C) calculate the stresses at a plane with $\alpha = 20$ *.*

then check your answer using mohr's circle.



$f_x = 1600\ kg/Cm^2$ شد $\qquad f_y = -400 kg/Cm^2$ ضغط

$q = 300\ kg/Cm^2 \Longrightarrow$ نأخذ اشارة الـ q الرأسية و تكون بالـ $+Ve$ لان دوران الـ q الرأسية مع عقارب الساعة

1)Principle Normal stresses and their planes

$$f_{max.} = \left[\frac{f_x + f_y}{2}\right] + \sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$f_{max.} = \left[\frac{1600 - 400}{2}\right] + \sqrt{\left[\frac{1600 + 400}{2}\right]^2 + 300^2}$$

$$f_{max.} = 600 + 1044 = 1644\ kg/Cm^2$$

$$f_{min.} = \left[\frac{f_x + f_y}{2}\right] - \sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$f_{min.} = \left[\frac{1600 - 400}{2}\right] - \sqrt{\left[\frac{1600 + 400}{2}\right]^2 + 300^2}$$

$$f_{min.} = 600 - 1044 = -444\ kg/Cm^2$$

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* المستوى الذى عليه $f_{max.}$ or $f_{min.}$ لا يكون عليه Shear stresses $q = 0$

و لايجاد مستوى الـ $f_{max.}$ & $f_{min.}$ نحسب الـ θ

$$\tan 2\theta = \frac{-2q}{f_x - f_y} = \frac{-2(300)}{1600-(-400)} = -0.30$$

$$2\theta = -16.7^\circ \qquad \boxed{\theta = -8.35^\circ}$$

* للوصول للمستوى الذى عليه $f_{max.}$ تقاس الـ θ من المستوى اللى عليه الكبير من الـ f_x or f_y .
* المستوى الذى عليه $f_{min.}$ يكون عمودى على المستوى اللى عليه الـ $f_{max.}$ أو نقيس الزاوية θ من المستوى اللى عليه الاصغر من الـ f_x or f_y .

2)Principle Shear stresses and their planes

$$q_{max.} = + \sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$q_{max.} = + \sqrt{\left[\frac{1600+400}{2}\right]^2 + 300^2} = 1044\ kg/Cm^2$$

$$q_{min.} = - \sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$q_{min.} = - \sqrt{\left[\frac{1600+400}{2}\right]^2 + 300^2} = -1044\ kg/Cm^2$$

* المستوى الذى عليه $q_{max.}$ or $q_{min.}$ يكون عليه Shear stresses تساوى $f_{av.}$

$$f_{av.} = \frac{f_x + f_y}{2} = \frac{1600-400}{2} = 600\ kg/Cm^2$$

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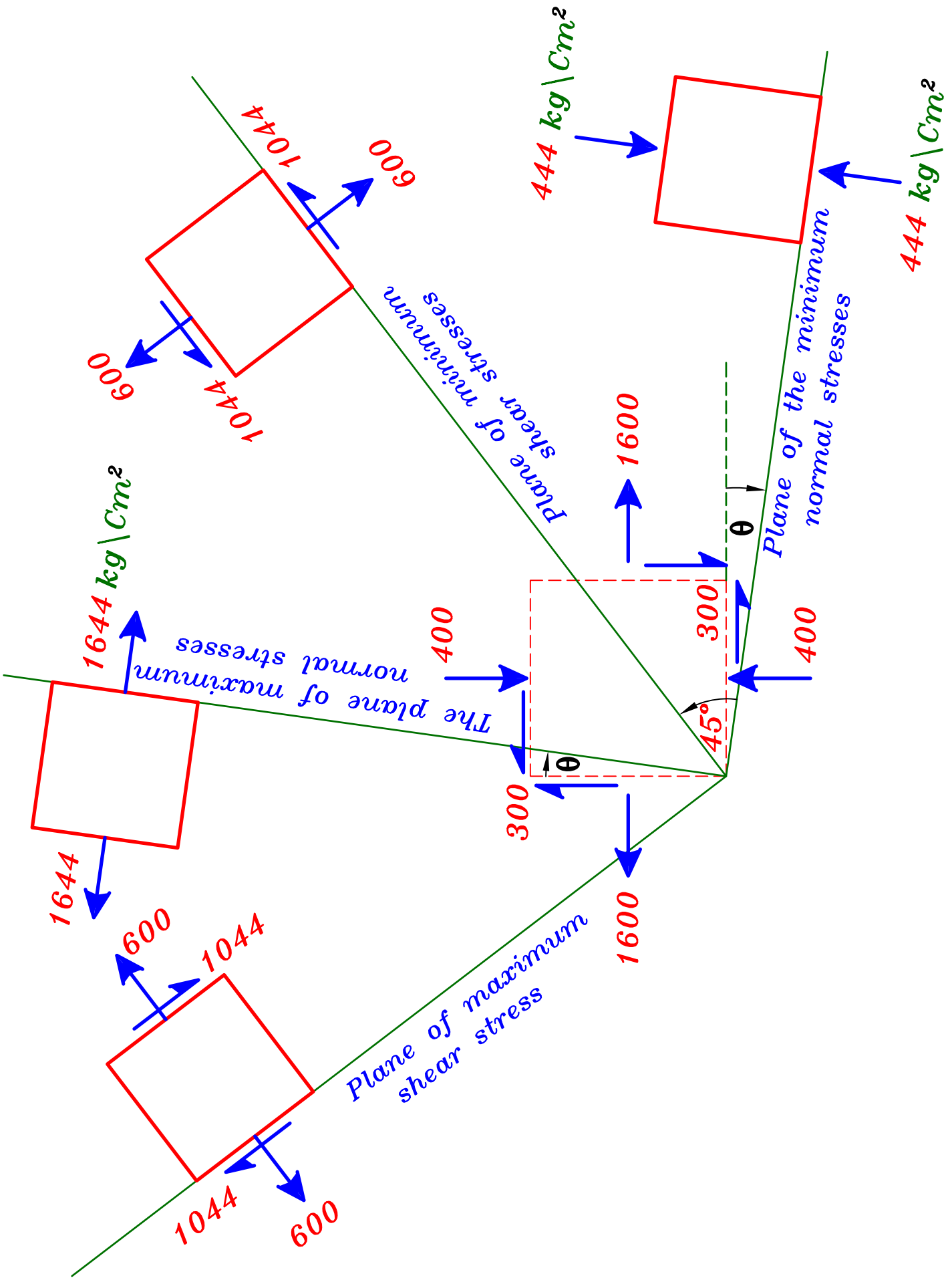

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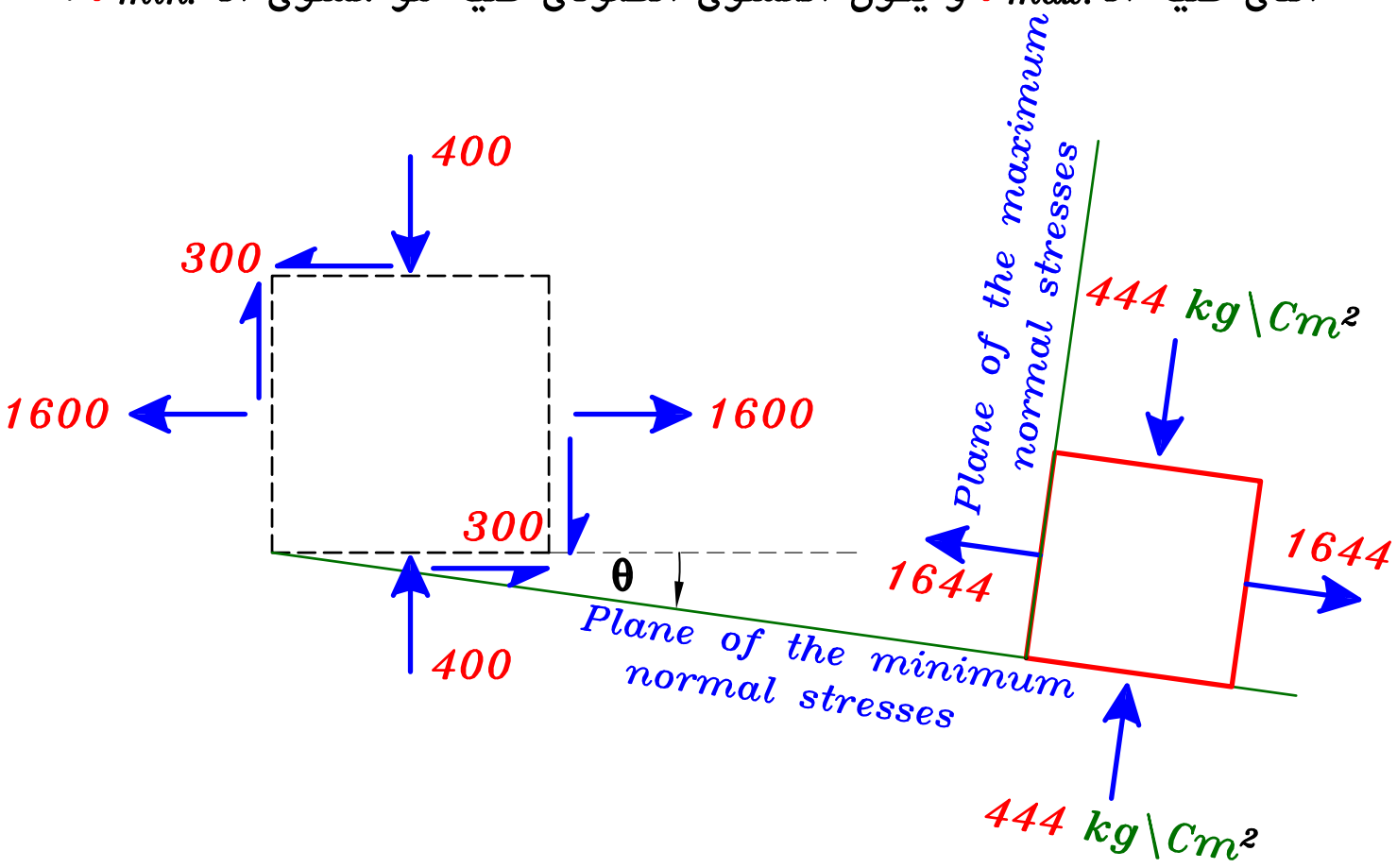
* و لكن من الممكن رسم الـ maximum&minimum normal stresses على نفس الرسمة .
* و من الممكن أيضا رسم الـ maximum&minimum shear stresses على نفس الرسمة .

١ - نرسم المكعب الاصلى بخط dotted .

٢ - نلف المكعب بزاوية θ و فى هذه المسألة الـ θ بالـ Ve − أى نلف مع عقارب الساعة و يصبح المستوى الذى كان عليه الاكبر من f_x & f_y هو المستوى الذى عليه الـ $f_{max.}$ و يكون المستوى العمودى عليه هو مستوى الـ $f_{min.}$.



٣ - نلف المكعب بزاوية 45 من الوضع الجديد الذى فيه مستوى الـ maximum و الـ minimum للـ normal stresses و بعد دوران المكعب يكون المستوى الذى عليه الـ maximum shear stresses هو المستوى اللى كان عليه الـ maximum normal stresses و المستوى اللى عليه الـ minimum shear stresses هو المستوى اللى كان عليه الـ min. normal stresses .
و الزاوية الـ 45 تكون عكس عقارب الساعة

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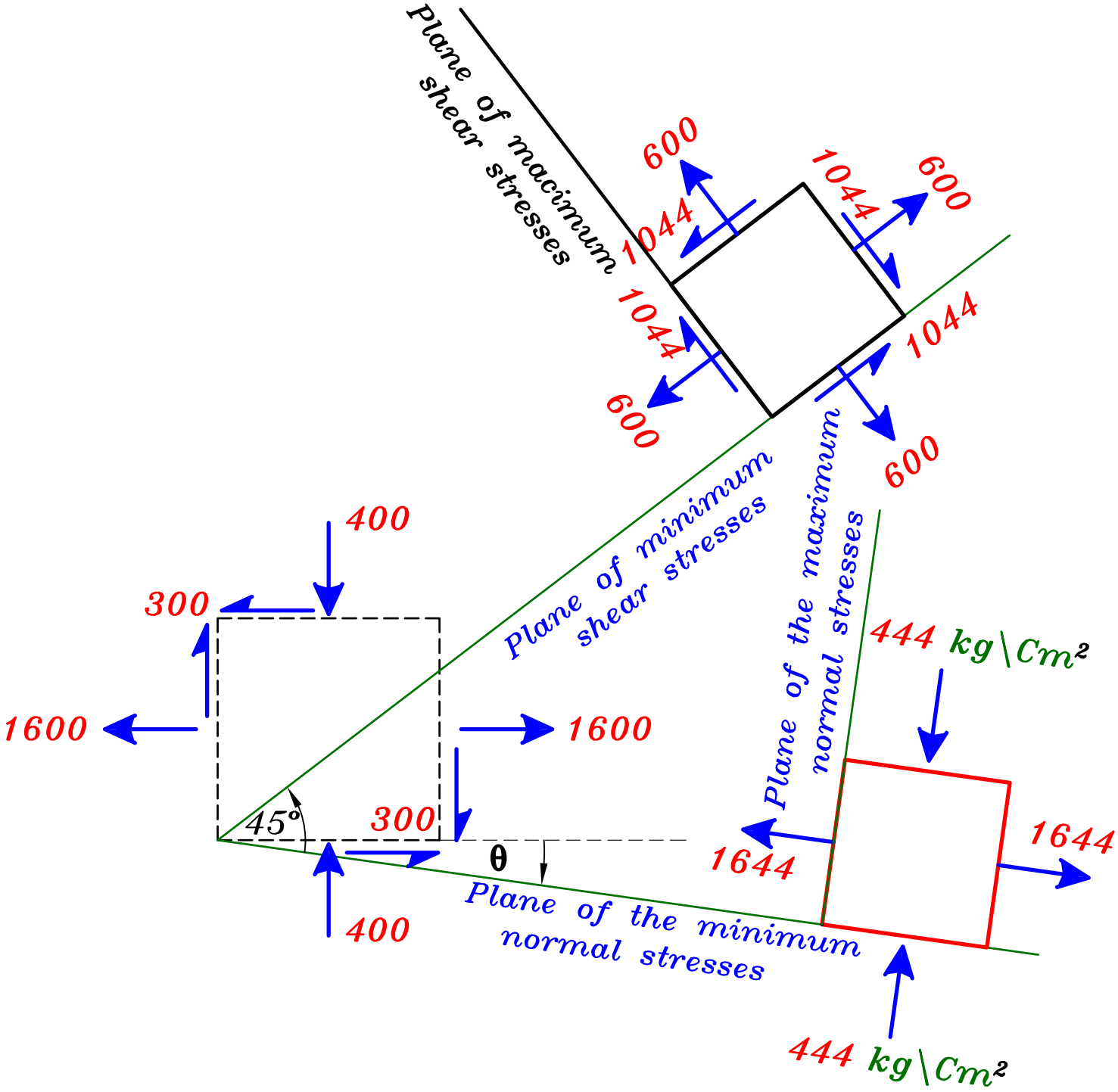

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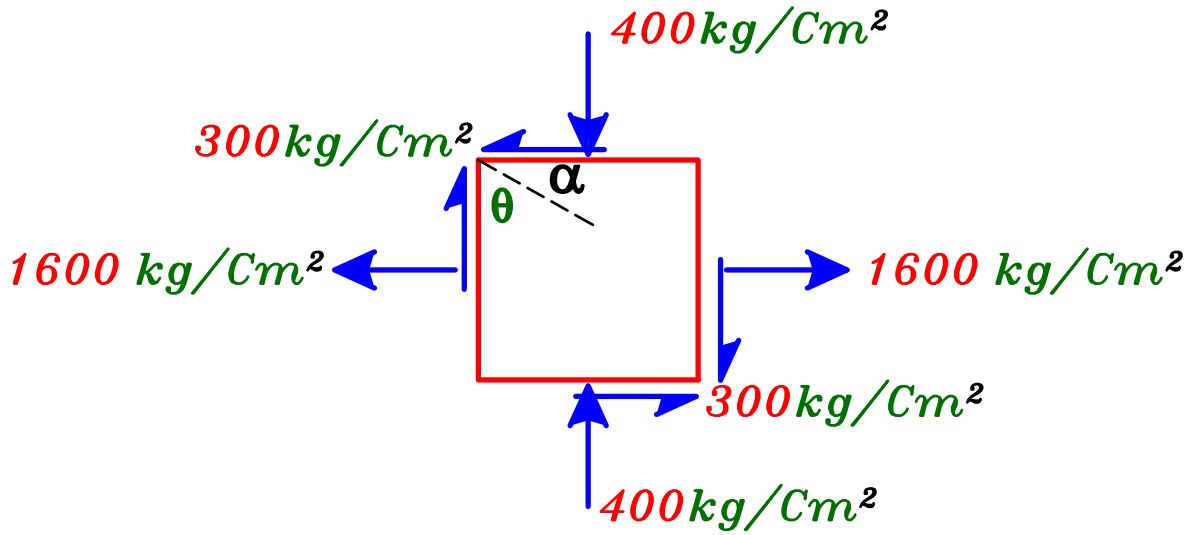
3)Stresses at plane with $\alpha = 20$.



* لابد أن تكون الزاوية θ مقاسة من المحور الرأسى الى المستوى المراد حساب الـStresses عليه فى اتجاه عكس عقارب الساعة.

$$\theta = 90 - 20 = 70$$

$$f_\theta = \left[\frac{f_x + f_y}{2}\right] + \left[\frac{f_x - f_y}{2}\right] \cos 2\theta - q \sin 2\theta$$

$$f_\theta = \left[\frac{1600 - 400}{2}\right] + \left[\frac{1600 + 400}{2}\right] \cos 140 - 300 \sin 140$$

$$f_\theta = -358.88 \ kg/Cm^2$$

$$q_\theta = \left[\frac{f_x - f_y}{2}\right] \sin 2\theta + q \cos 2\theta$$

$$q_\theta = \left[\frac{1600 + 400}{2}\right] \sin 140 + 300 \cos 140$$

$$q_\theta = 412.97 \ kg/Cm^2$$

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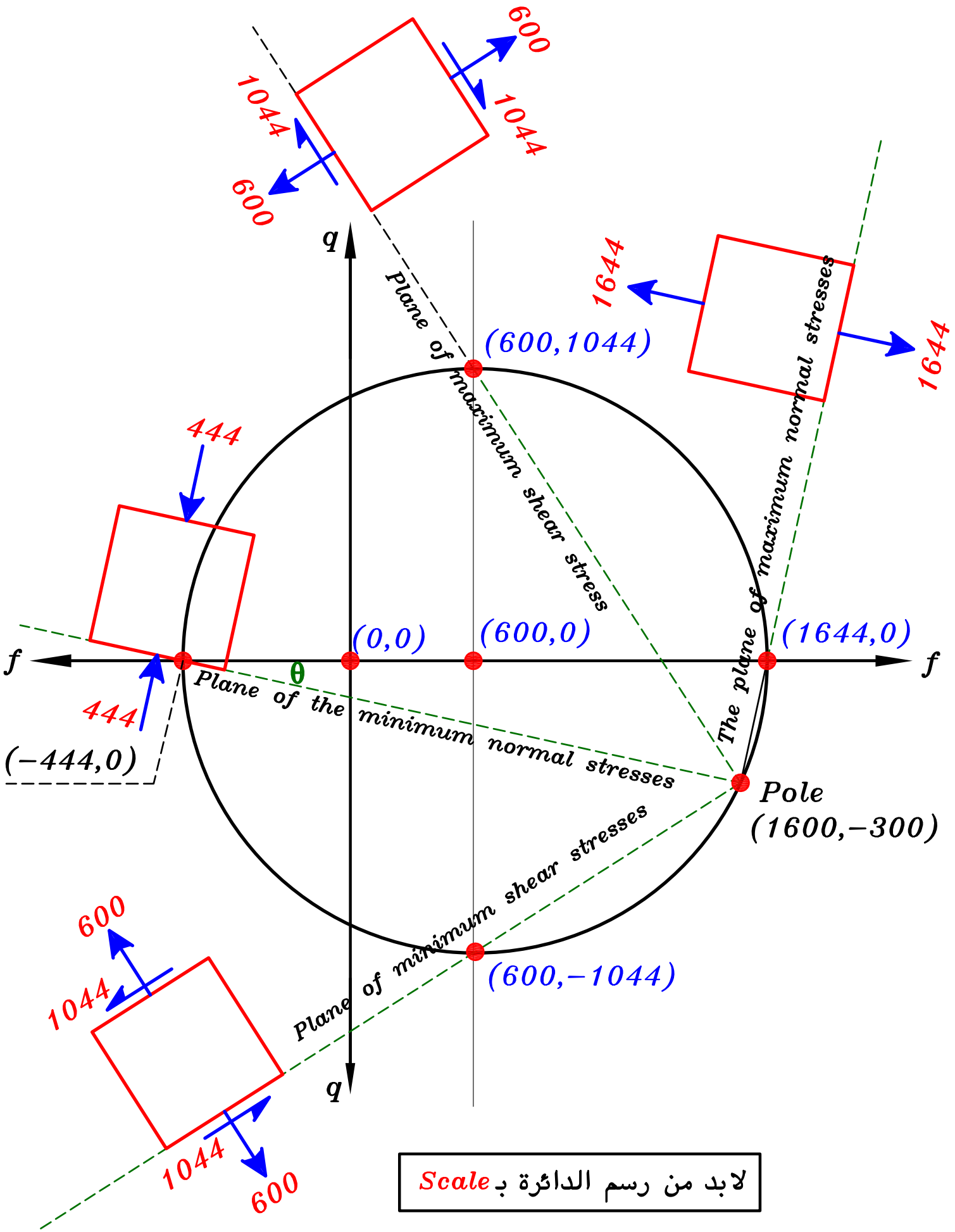
mohr's circle for stresses



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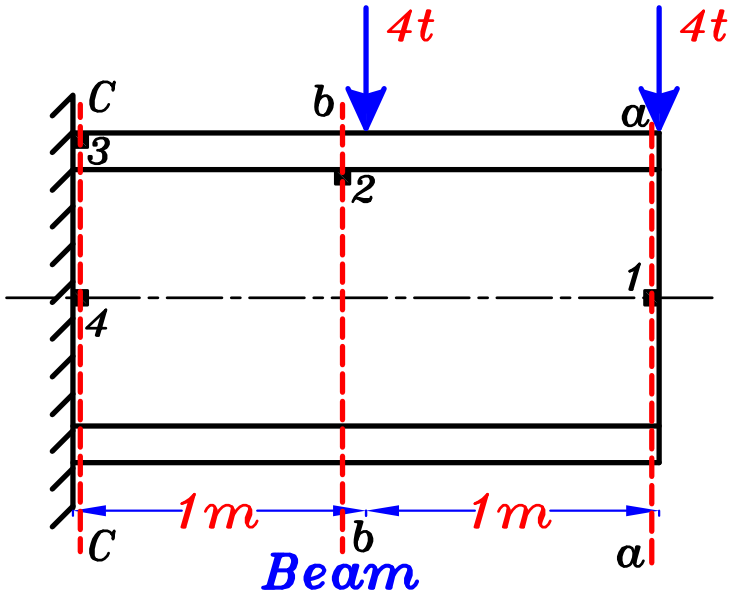
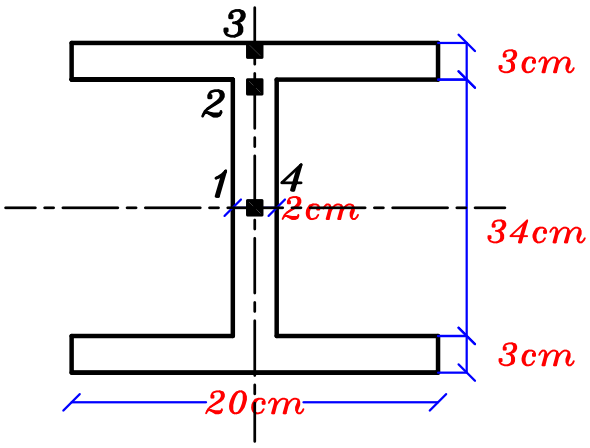
Example:

For the shown beam , it is required to get the principal normal & shear stresses & their planes at the shown points.



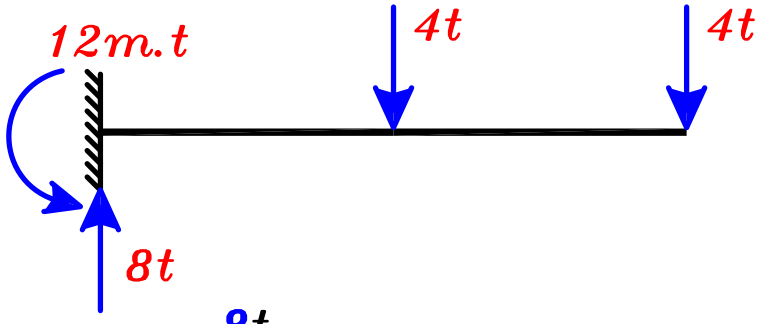
Beam

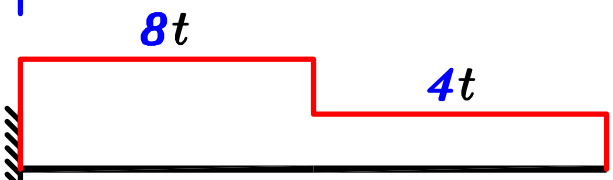


Beam cross section

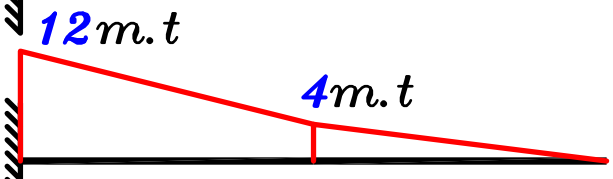
فكرة المسألة هى كيفية حساب الـ *Normal & Shear Stresses* عند كل نقطة من النقط لانه بمعرفة الـ *Normal & Shear Stresses* يمكننا حساب الـ *Principal stresses* و مستوياتها .

و لحساب الـ *stresses* نحتاج الى حساب الـ *Straining actions* على الكمرة

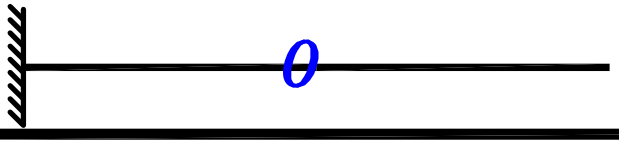




S.F.D



B.M.D



N.F.D

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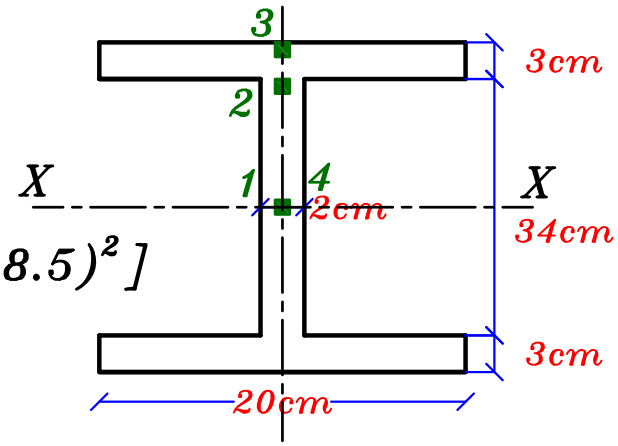
ثم نحسب الـ *Properties of area* لقطاع الكمرة

Properties of area



$$A = 20 * 3 * 2 + 34 * 2$$

$$= 188\ Cm^2$$

$$I_x = 2\left[(20 * 3^3/12) + (20x\ 3) * (18.5)^2 \right] + \left[(2 * 34^3/12) \right]$$

$$= 47710\ Cm^4$$

Point (1) at sec a–a

straining actions

$N = 0$ من الـ *N.F.D*

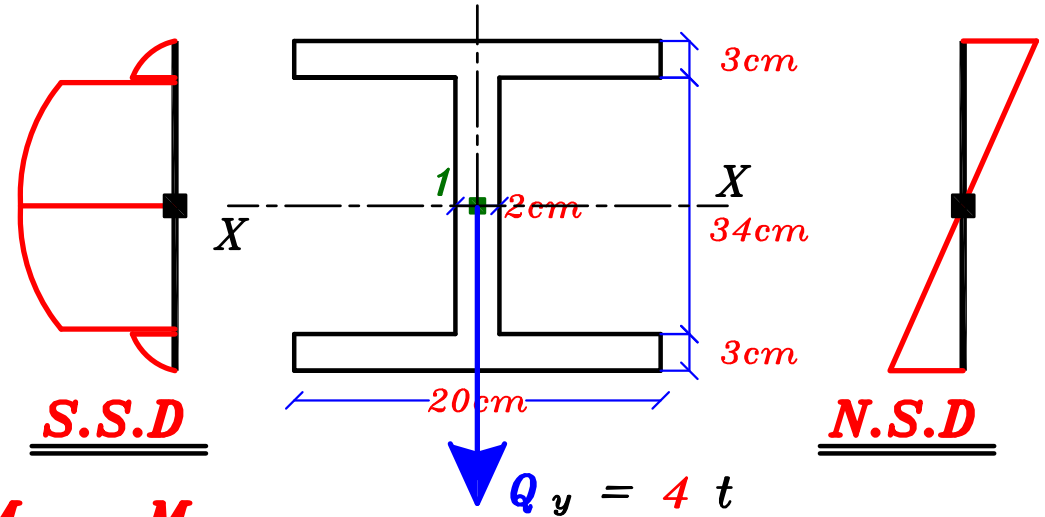
$Q_y = 4\ t \downarrow$ من الـ *S.F.D*

$M_x = 0\ m.t$ من الـ *B.M.D*

$M_y = 0$

$Q_x = 0$

$M_z = 0$



Stresses

$$\# f = \pm \frac{N}{A} \pm \frac{M_x}{I_x} y \pm \frac{M_y}{I_y} x$$

معادلة الـ *Normal stress* كالتالى

و حيث أن الـ $N = 0$ & $M_y = 0$

تصبح المعادلة كالتالى

$$f = \pm \frac{M_x}{I_x} y = 0$$

$$\# q = \frac{Q}{I} x \frac{S}{b}$$

$$= \frac{4}{47710} * \frac{(20x\ 3) * (18.5) + (17x\ 2) * (8.5)}{2}$$

$$= 0.0586\ t\backslash Cm^2$$

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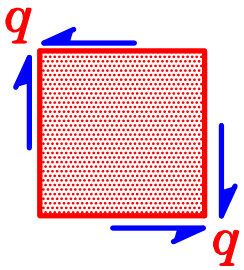
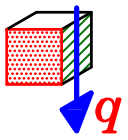
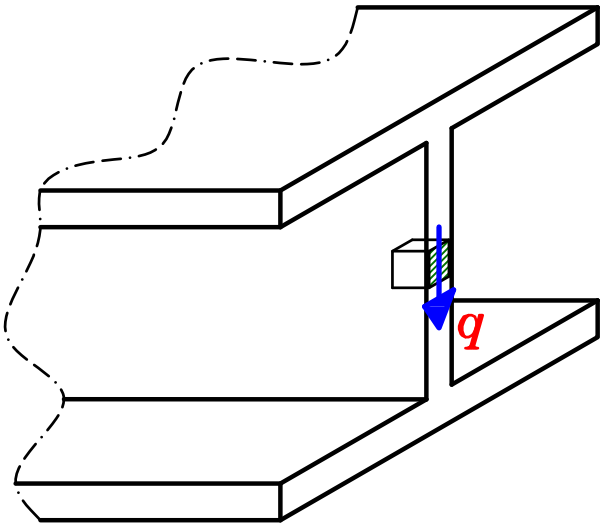
For principal stresses







نختار الوجه الذى يظهر فيه
الـ Normal stress عمودى على أحد
أضلاعه و الـ Shear stress موازى
لاحد أضلاعه .

$f_x = 0$

$f_y = 0$

$q = +0.0586\ t/Cm^2$

1)Principle Normal stresses and their planes

$$f_{max.} = [\frac{f_x + f_y}{2}] + \sqrt{[\frac{f_x - f_y}{2}]^2 + q^2}$$

$$f_{max.} = [\frac{0 + 0}{2}] + \sqrt{[\frac{0 - 0}{2}]^2 + 0.0586^2}$$

$$f_{max.} = 0.0586\ t/Cm^2$$

$$f_{min.} = [\frac{f_x + f_y}{2}] - \sqrt{[\frac{f_x - f_y}{2}]^2 + q^2}$$

$$f_{min.} = [\frac{0 + 0}{2}] - \sqrt{[\frac{0 - 0}{2}]^2 + 0.0586^2}$$

$$f_{min.} = -0.0586\ t/Cm^2$$

* المستوى الذى عليه $f_{max.}$ or $f_{min.}$ لا يكون عليه Shear stresses

$$\boxed{q = 0}$$

$$\tan 2\theta = \frac{-2q}{f_x - f_y} = \frac{-2(0.0586)}{0 - 0}$$

$$2\theta = -90^\circ \qquad \boxed{\theta = -45^\circ}$$

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2)Principle Shear stresses and their planes

$$q_{max.} = + \sqrt{[\frac{f_x - f_y}{2}]^2 + q^2}$$

$$q_{max.} = + \sqrt{[\frac{0-0}{2}]^2 + 0.0586^2} = 0.0586\ t/Cm^2$$

$$q_{min.} = - \sqrt{[\frac{f_x - f_y}{2}]^2 + q^2}$$

$$q_{min.} = - \sqrt{[\frac{0-0}{2}]^2 + 0.0586^2} = -0.0586\ t/Cm^2$$

* المستوى الذى عليه $q_{max.}$ or $q_{min.}$ يكون عليه Shear stresses تساوى $f_{av.}$

$$f_{av.} = \frac{f_x + f_y}{2} = \frac{0+0}{2} = 0$$

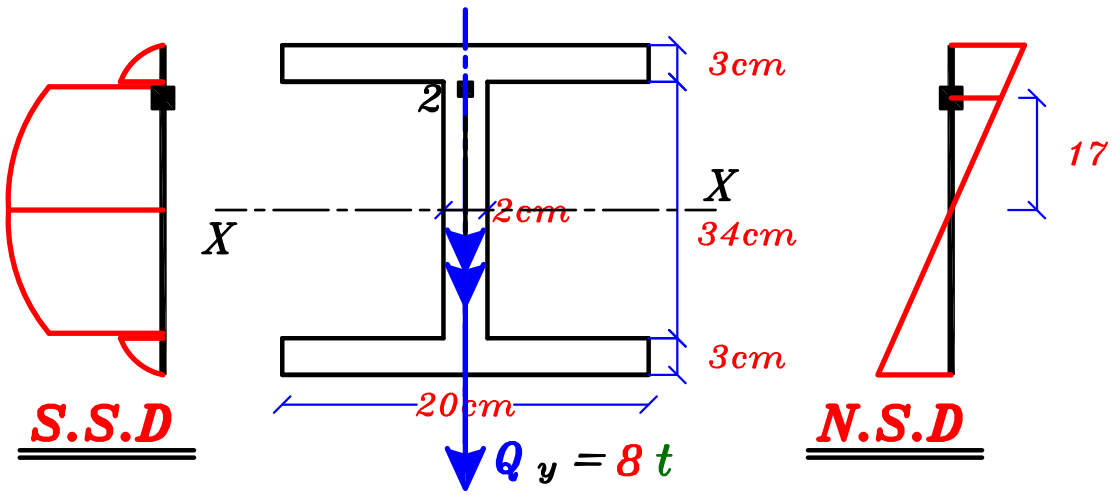
Point (2) at sec b–b

straining actions

$N = 0$ من الـ N.F.D

$Q_y = 8\ t \downarrow$ من الـ S.F.D

$M_x = 4\ m.t \downarrow$ من الـ B.M.D



Stresses

$$f = + \frac{M_x}{I_x} y$$

$$= \frac{4 * 100}{47710} * 17 = 0.142\ t\backslash Cm^2$$ شد

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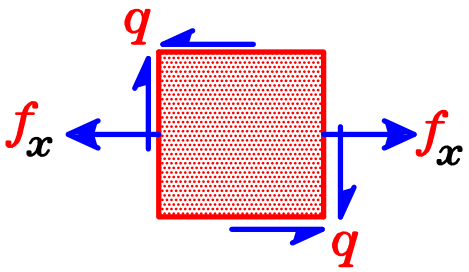
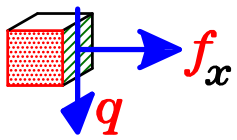
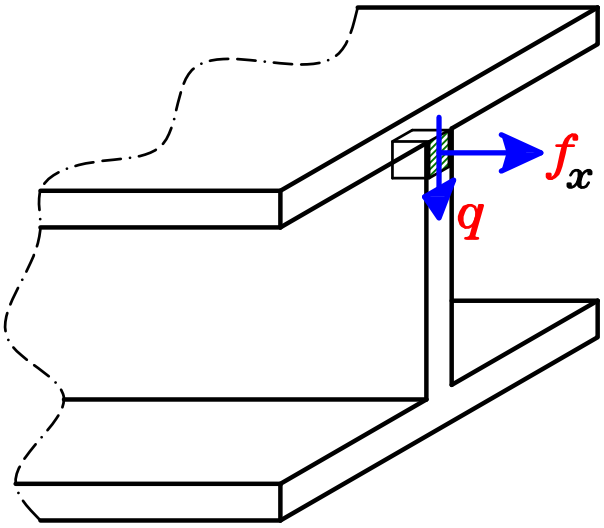
$$\# \; q = \frac{Q}{I} \; x \; \frac{S}{b}$$

$$= \frac{8}{47710} * \frac{(20 x \; 3) * (18.5)}{2} = 0.093 \; t\backslash Cm^2$$

For principal stresses







$f_x = 0.142 \; t\backslash Cm^2$

$f_y = 0$

$q = +0.093 \; t/Cm^2$

نختار الوجه الذى يظهر فيه
الـ Normal stress عمودى على أحد
أضلاعه و الـ Shear stress موازى
لاحد أضلاعه .

1)Principle Normal stresses and their planes

$$f_{\substack{max. \\ min.}} = \left[\frac{f_x + f_y}{2} \right] \pm \sqrt{\left[\frac{f_x - f_y}{2} \right]^2 + q^2}$$

$$f_{\substack{max. \\ min.}} = \left[\frac{0.142 + 0}{2} \right] \pm \sqrt{\left[\frac{0.142 - 0}{2} \right]^2 + 0.093^2}$$

$f_{max.} = 0.188 \; t/Cm^2$

$f_{min.} = -0.046 \; t/Cm^2$

$\boxed{q = 0}$

$$\tan 2\theta = \frac{-2q}{f_x - f_y} = \frac{-2(0.093)}{0.142 - 0}$$

$2\theta = -52.6^\circ$ $\qquad \boxed{\theta = -26.3^\circ}$

2)Principle Shear stresses and their planes

$$q_{\substack{max. \\ min.}} = \pm \sqrt{\left[\frac{f_x - f_y}{2} \right]^2 + q^2}$$

$$q_{\substack{max. \\ min.}} = \pm \sqrt{\left[\frac{0.142 - 0}{2} \right]^2 + 0.093^2}$$

$q_{max.} = 0.117 \; t/Cm^2$ $\qquad q_{min.} = -0.117 \; t/Cm^2$

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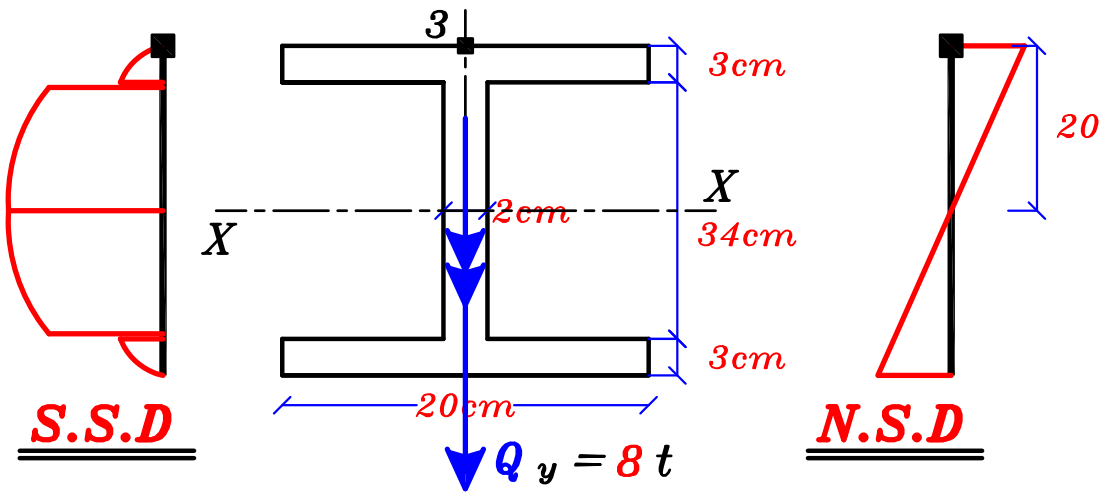
Point (3) at sec C–C

straining actions

$N = 0$ من الـ N.F.D

$Q_y = 8\ t \downarrow$ من الـ S.F.D

$M_x = 12\ m.t \downarrow$ من الـ B.M.D



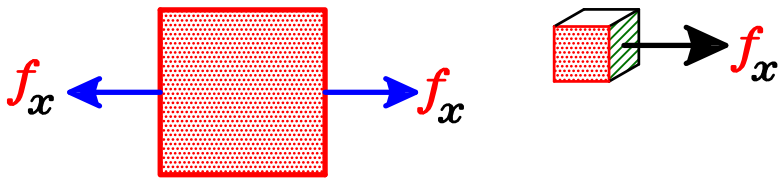
Stresses

$$\# f = + \frac{M_x}{I_x} y$$

$$= \frac{12 * 100}{47710} * 20 = 0.50\ t\backslash Cm^2$$ شد

$$\# q = \frac{Q}{I} x \frac{S}{b} = 0$$

For principal stresses

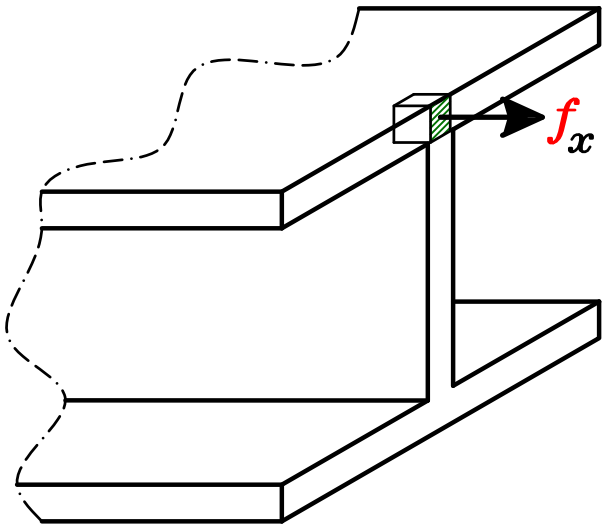


$f_x = 0.50\ t\backslash Cm^2$

$f_y = 0$

$q = 0$

نختار الوجه الذى يظهر فيه الـ Normal stress عمودى على أحد أضلاعه و الـ Shear stress موازى لاحد أضلاعه .



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1)Principle Normal stresses and their planes

$$f_{\substack{max.\\min.}} = \left[\frac{f_x + f_y}{2}\right] \pm \sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$f_{\substack{max.\\min.}} = \left[\frac{0.50 + 0}{2}\right] \pm \sqrt{\left[\frac{0.50 - 0}{2}\right]^2}$$

$$f_{max.} = 0.50 \ t/Cm^2$$

$$f_{min.} = 0$$

$$\boxed{q = 0}$$

$$\tan 2\theta = \frac{-2q}{f_x - f_y} = \frac{-2(0)}{0.50 - 0}$$

$$2\theta = 0^\circ \qquad \boxed{\theta = 0^\circ}$$

2)Principle Shear stresses and their planes

$$q_{\substack{max.\\min.}} = \pm \sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$q_{\substack{max.\\min.}} = \pm \sqrt{\left[\frac{0.50 - 0}{2}\right]^2}$$

$$q_{max.} = 0.25 \ t/Cm^2 \qquad q_{min.} = -0.25 \ t/Cm^2$$

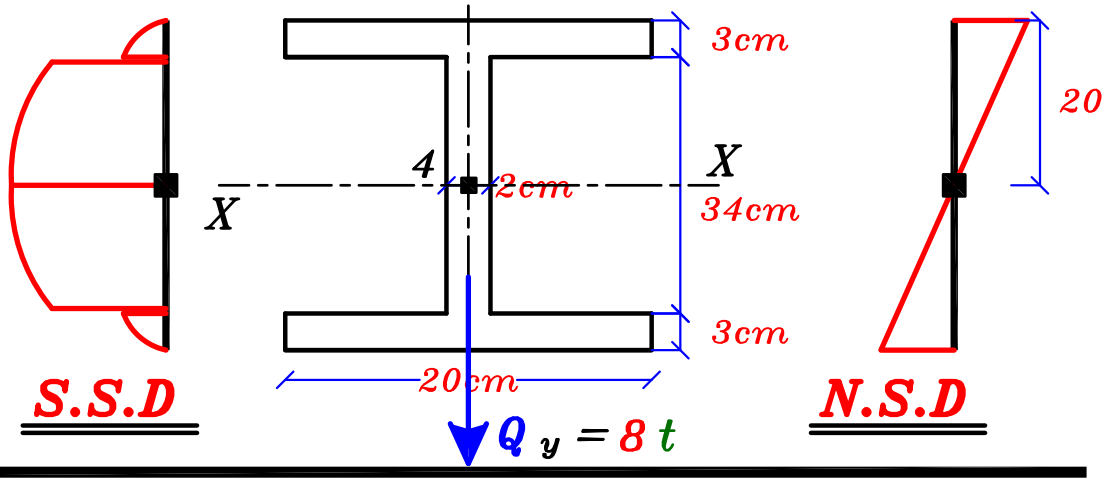
Point (4) at sec C–C

straining actions

$N = 0$ من الـ N.F.D

$Q_y = 8\ t \downarrow$ من الـ S.F.D

$M_x = 12\ m.t \downarrow$ من الـ B.M.D



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Stresses

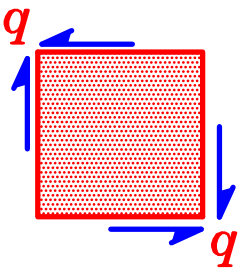
$$\# f = + \frac{M_x}{I_x} y = 0$$

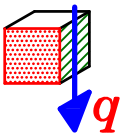
$$\# q = \frac{Q}{I} x \frac{S}{b}$$

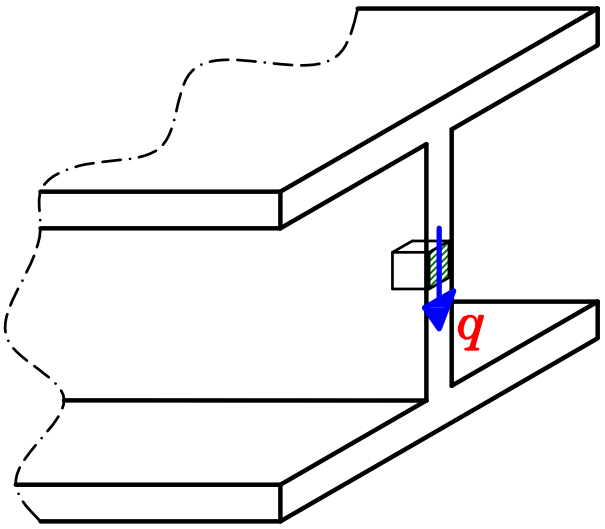
$$= \frac{8}{47710} * \frac{(20x3)*(18.5)+(17x2)*(8.5)}{2}$$

$$= 0.117\ t\backslash Cm^2$$

For principal stresses







$f_x = 0$

$f_y = 0$

$q = +0.117\ t/Cm^2$

نختار الوجه الذى يظهر فيه
الـ *Normal stress* عمودى على أحد
أضلاعه و الـ *Shear stress* موازى
لاحد أضلاعه .

1)Principle Normal stresses and their planes

$$f_{\substack{max.\\min.}} = \left[\frac{f_x + f_y}{2}\right] \pm \sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$f_{\substack{max.\\min.}} = \left[\frac{0+0}{2}\right] \pm \sqrt{\left[\frac{0-0}{2}\right]^2 + 0.117^2}$$

$$f_{max.} = 0.117\ t/Cm^2$$

$$f_{min.} = -0.117\ t/Cm^2$$

$q = 0$

$$\tan 2\theta = \frac{-2q}{f_x - f_y} = \frac{-2(0.117)}{0 - 0}$$

$$2\theta = -90^\circ \qquad \boxed{\theta = -45^\circ}$$

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2)Principle Shear stresses and their planes

$$q_{\substack{max.\\min.}} = \pm\sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$q_{\substack{max.\\min.}} = \pm\sqrt{\left[\frac{0 - 0}{2}\right]^2 + 0.117^2}$$

$$q_{max.} = 0.117\ t/Cm^2 \qquad q_{min.} = -0.117\ t/Cm^2$$

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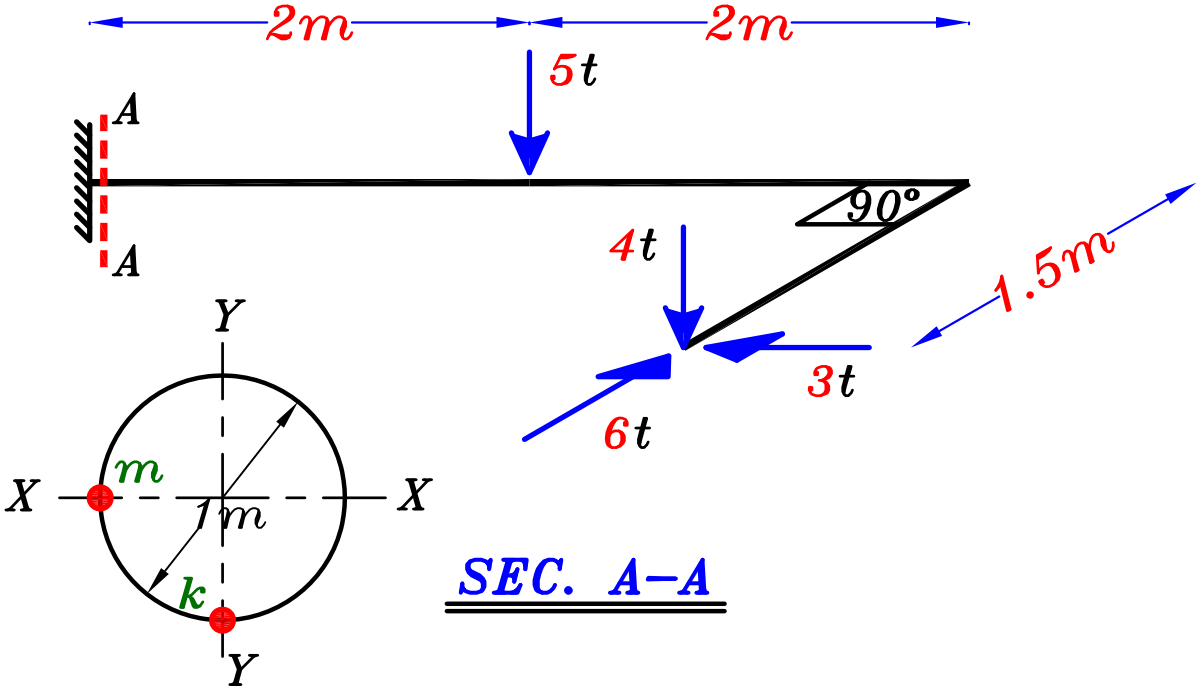
Example:

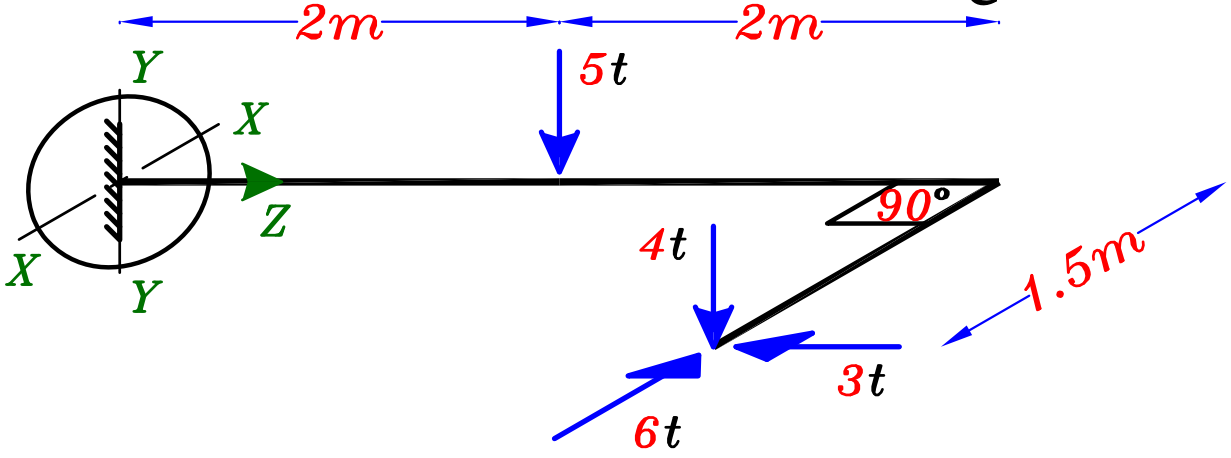
For the shown beam , it is required to get the principal normal & shear stresses & their planes at points (m,k).



SEC. A–A

فكرة المسألة هى كيفية حساب الـ *Normal & Shear Stresses* عند كل نقطة من النقط لانه بمعرفة الـ *Normal & Shear Stresses* يمكننا حساب الـ *Principal stresses* و مستوياتها .

و لحساب الـ *stresses* نحتاج الى حساب الـ *Straining actions* على الكمرة



straining actions

$N = 3\ t$ (Comp.)

$Q_y = 9\ t \downarrow$

$Q_x = 6\ t \nearrow$

$M_x = 5 * 2 + 4 * 4 = 26\ m.t \downarrow$

$M_y = 6 * 4 - 3 * 1.5 = 19.5\ m.t \nearrow$

$M_z = 4 * 1.5 = 6\ m.t \circlearrowleft$

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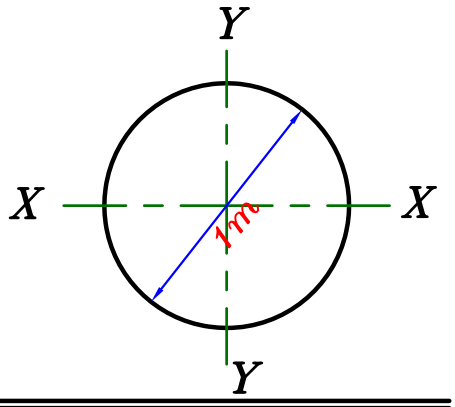
Properties of area

نحتاج الى حساب الـ $I_x \& A$ من اجل الـ Normal Stresses كما نحتاج الى حساب الـ I_P من اجل الـ Shear Stresses لوجود M_t.

$$A = \pi r^2 = \pi (0.5)^2 = 0.785\ m^2$$

$$I_x = I_y = \frac{\pi}{4} r^4 = \frac{\pi}{4} (0.5)^4 = 0.049\ m^4$$

$$I_P = I_x + I_y = 0.098\ m^4$$



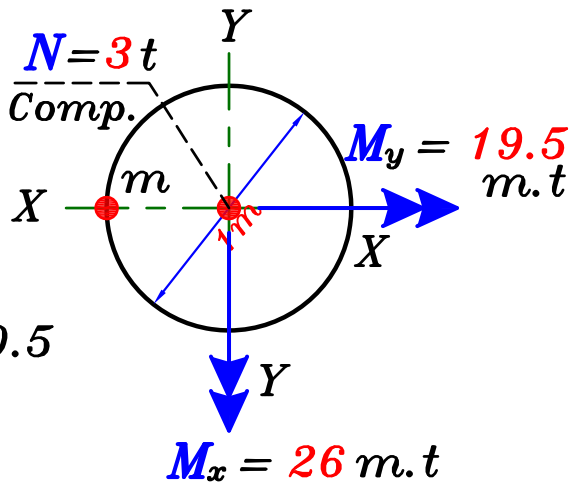
Point (m) at sec A–A

Normal stresses

$$\# f = \pm \frac{N}{A} \pm \frac{M_x}{I_x} y \pm \frac{M_y}{I_y} x$$

$$f_m = -\frac{3}{0.785} + \frac{26}{0.049} * 0 + \frac{19.5}{0.049} * 0.5$$

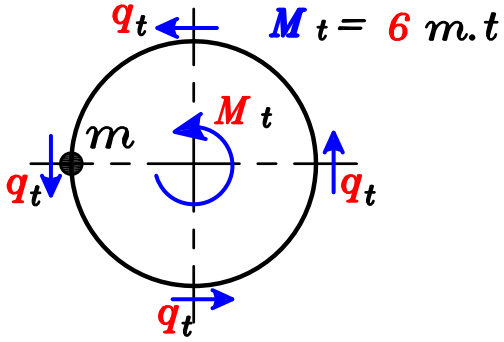
$$= +195\ t\backslash m^2$$ شد



Shear stresses

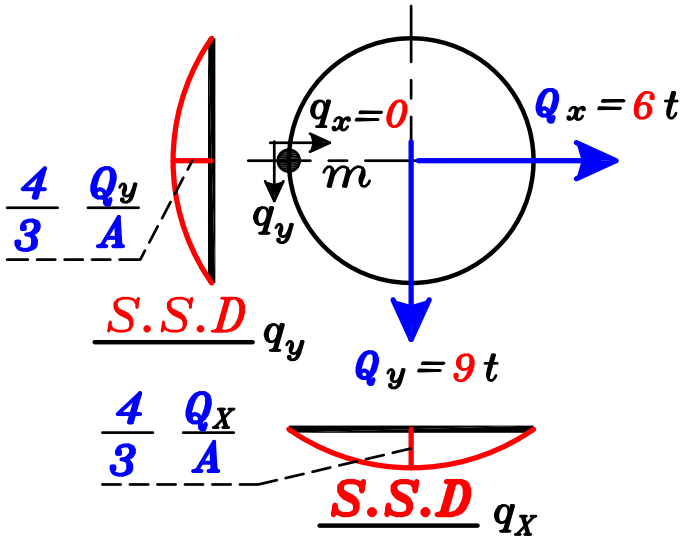
نحسب الـ Shear Stresses نتيجة $Q_x \& Q_y$ ثم نحسب الـ Shear Stresses نتيجة M_t ثم نجمعهم معا.

Shear stresses due to Torsion



قيمة الـ q_t متساوية عند كل النقط الموجودة على محيط الدائرة لان لها نفس الـ radius

Shear stresses due to forces



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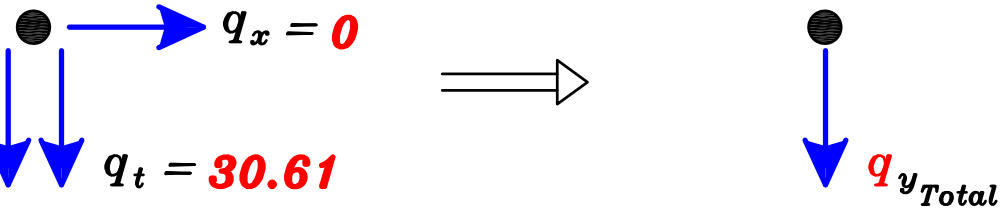
$$\# \; q_y = \frac{Q_y}{I} \; x \; \frac{S}{b} = \frac{4}{3} \; x \; \frac{Q_y}{A}$$

$$= \frac{4}{3} * \frac{9}{0.785} = 15.28 \; t\backslash m^2 \downarrow$$

اتجاه ال Shear Stress هو نفس اتجاه ال Shear force

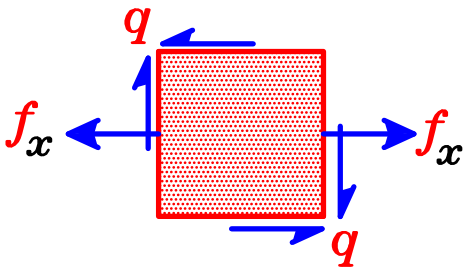
$$\# \; q_x = 0$$

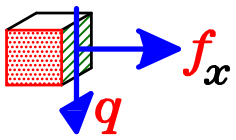
$$\# \; q_t = \frac{M_t}{I_P} \; x \; R = \frac{6}{0.098} * 0.5 = 30.61 \; t\backslash m^2 \downarrow$$

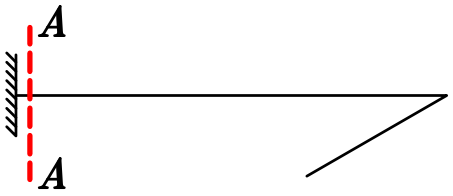


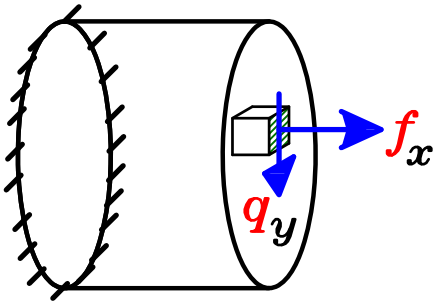
$$\# \; q_{y_{Total}} = q_y + q_t = 45.89 \; t\backslash m^2$$

For principal stresses









$f_x = +195 \; t\backslash m^2$

$f_y = 0$

$q = +45.89 \; t\backslash m^2$

نختار الوجه الذى يظهر فيه ال Normal stress عمودى على أحد أضلاعه و ال Shear stress موازى لاحد أضلاعه .

1)Principle Normal stresses and their planes

$$f_{\substack{max. \\ min.}} = \left[\frac{f_x + f_y}{2} \right] \pm \sqrt{\left[\frac{f_x - f_y}{2} \right]^2 + q^2}$$

$$f_{\substack{max. \\ min.}} = \left[\frac{195 + 0}{2} \right] \pm \sqrt{\left[\frac{195 - 0}{2} \right]^2 + 45.89^2}$$

$f_{max.} = 205 \; t\backslash m^2$

$f_{min.} = -10 \; t\backslash m^2$

$q = 0$

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$$\tan 2\theta = \frac{-2q}{f_x - f_y} = \frac{-2(45.89)}{195 - 0}$$

$$2\theta = -25.2^\circ \qquad \boxed{\theta = -12.6^\circ}$$

2)Principle Shear stresses and their planes

$$q_{\substack{max.\\min.}} = \pm\sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$q_{\substack{max.\\min.}} = \pm\sqrt{\left[\frac{195 - 0}{2}\right]^2 + 45.89^2}$$

$q_{max.} = 107.7\ t\backslash m^2$ $\qquad$ $q_{min.} = -107.7\ t\backslash m^2$

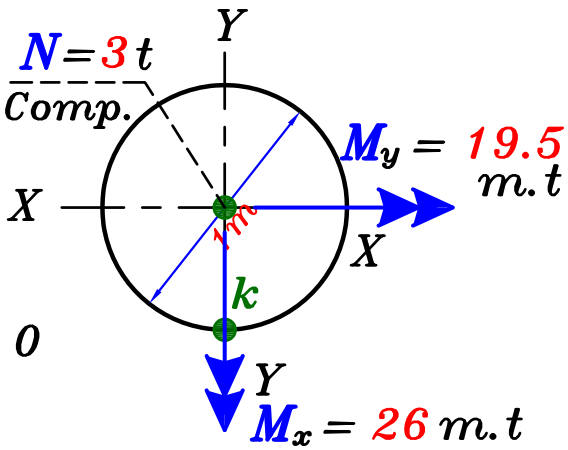
Point (k) at sec A–A

Normal stresses

$$\# f = \pm\frac{N}{A} \pm \frac{M_x}{I_x}y \pm \frac{M_y}{I_y}x$$

$$f_k = -\frac{3}{0.785} - \frac{26}{0.049} * 0.5 + \frac{19.5}{0.049} * 0$$

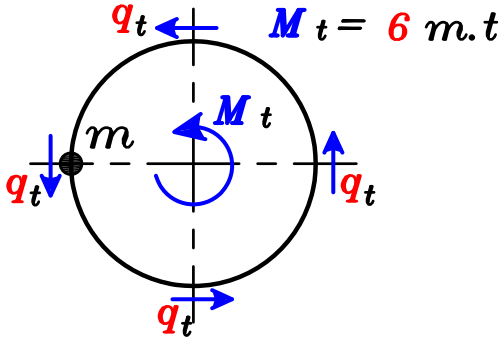
$$= -270\ t\backslash m^2 \text{ ضغط}$$



Shear stresses

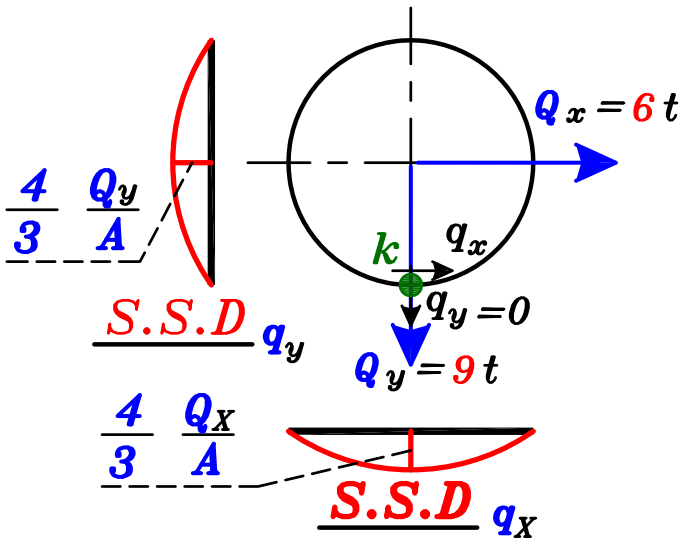
نحسب الـ Shear Stresses نتيجة Q_x & Q_y ثم نحسب الـ Shear Stresses نتيجة M_t ثم نجمعهم معا .

Shear stresses due to Torsion



قيمة الـ q_t متساوية عند كل النقط الموجودة على محيط الدائرة لان لها نفس الـ radius

Shear stresses due to forces



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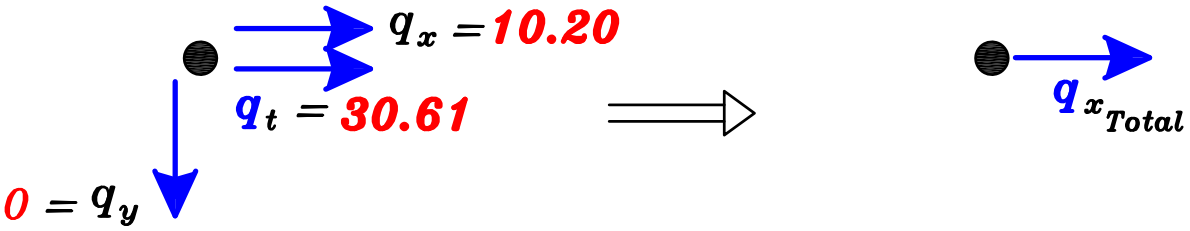
$$\# \; q_x = \frac{Q_x}{I} \; x \; \frac{S}{b} = \frac{4}{3} \; x \; \frac{Q_x}{A}$$

$$= \frac{4}{3} * \frac{6}{0.785} = 10.20 \; t\backslash m^2 \longrightarrow$$

اتجاه الـ Shear Stress هو نفس اتجاه الـ Shear force

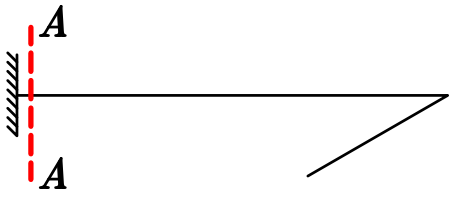
$$\# \; q_y = 0$$

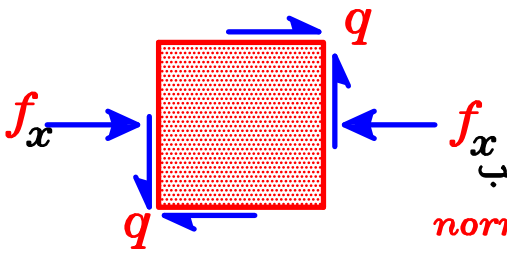
$$\# \; q_t = \frac{M_t}{I_P} \; x \; R = \frac{6}{0.098} * 0.5 = 30.61 \; t\backslash m^2 \longrightarrow$$

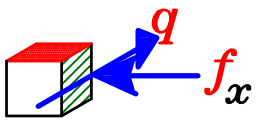


$$\# \; q_{x_{Total}} = q_x + q_t = 4080 \; t\backslash m^2$$

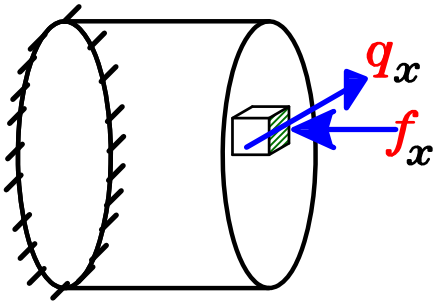
For principal stresses







الوجه الذى ندرسه من المكعب لابد أن يكون الـ normal stress عمودى على احد اضلاعه و الـ Shear stress موازى لاحد اضلاعه و لابد أن تظهر الـ Stresses على هذا الوجه خطوط وليست نقط



$f_x = -270 \; t\backslash m^2$ ضغط

$f_y = 0$

$q = -40.80 \; t\backslash m^2$ اشارة الـ q الرأسية سالبة لان الدوران عكس عقارب الساعة

1)Principle Normal stresses and their planes

$$f_{\substack{max. \\ min.}} = \left[\frac{f_x + f_y}{2} \right] \pm \sqrt{\left[\frac{f_x - f_y}{2} \right]^2 + q^2}$$

$$f_{\substack{max. \\ min.}} = \left[\frac{-270 + 0}{2} \right] \pm \sqrt{\left[\frac{-270 - 0}{2} \right]^2 + (-40.80)^2}$$

$f_{max.} = 6 \; t\backslash m^2$

$f_{min.} = -276 \; t\backslash m^2$

$q = 0$

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$$\tan 2\theta = \frac{-2q}{f_x - f_y} = \frac{-2(-40.80)}{-270 - 0}$$

$$2\theta = -16.8^\circ \qquad \boxed{\theta = -8.40^\circ}$$

2)Principle Shear stresses and their planes

$$q_{\substack{max.\\min.}} = \pm\sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$q_{\substack{max.\\min.}} = \pm\sqrt{\left[\frac{-270 - 0}{2}\right]^2 + (-40.80)^2}$$

$$q_{max.} = 141\ t\backslash m^2 \qquad q_{min.} = -141\ t\backslash m^2$$

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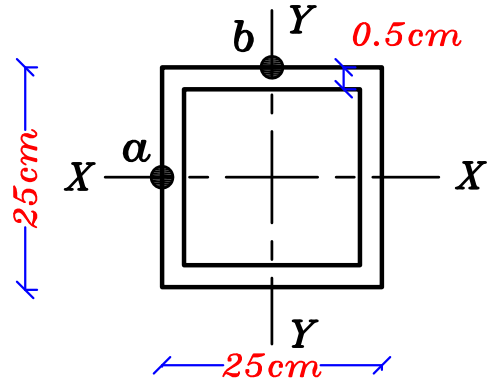
Example:

For the shown board , it is required to get the principal stresses and their planes at point (a,b) , knowing that the cross section for the post is a hollow square section with side 25 cm and thickness oh 0.5 cm.

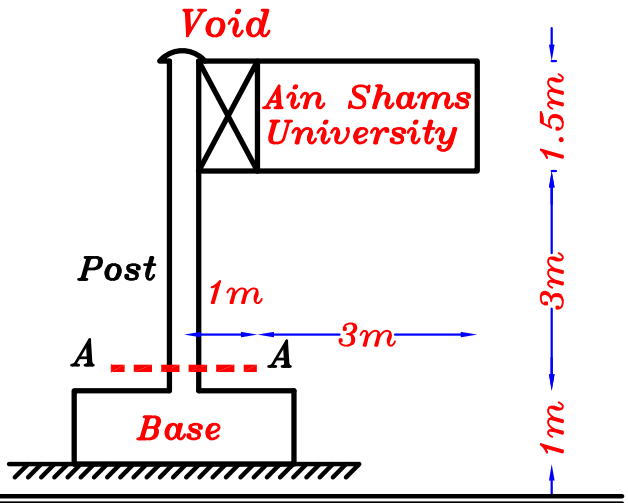
Wind pressure = 100 kg\m²

Own weight of board = 0.25 t

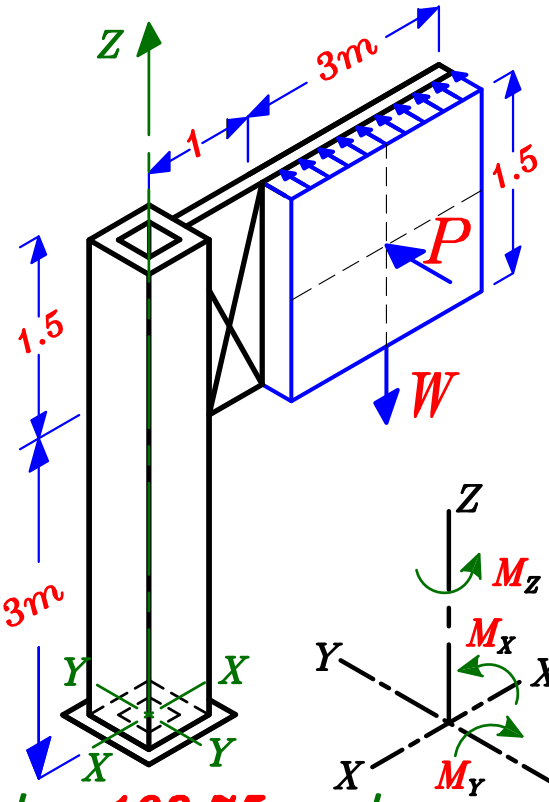
neglect own weight of post .



SEC. A–A



straining actions



$W = 0.25\ t$

P = wind pressure * Area

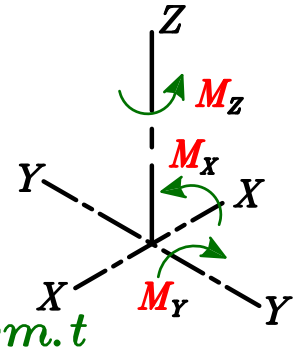
ونأخذ الـ Area الاكبر للوحة لانها هى التى تعطى التأثير الاكبر للـ wind أى ؤننا نأخذ الـ Area الامامية للوحة (3x1.5)

$P = 0.1\ t\backslash m^2 * (3x1.5) = 0.45\ t$

$N = 0.25\ t$ (Comp.)

$Q_y = 0.45\ t$ ↖

$Q_x = 0$



$M_X = 0.45 * (3 + 0.75) = 1.6875\ m.t = 168.75\ cm.t$

$M_Y = 0.25 * (1 + 1.50) = 0.625\ m.t \twoheadrightarrow = 62.5\ cm.t$

$M_Z = 0.45 * (1 + 1.50) = 1.125\ m.t \circlearrowleft = 112.5\ cm.t$

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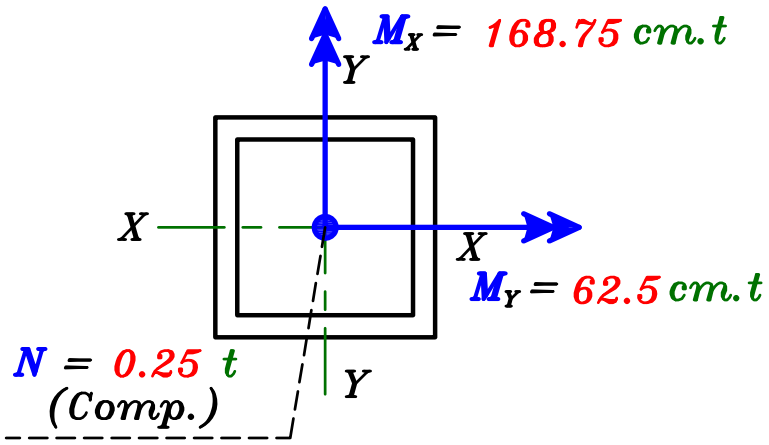
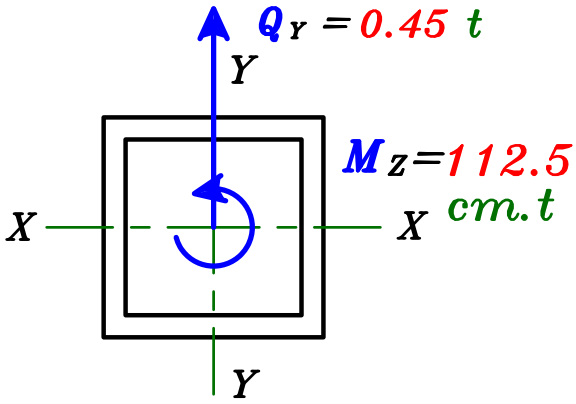


straining actions for normal stresses

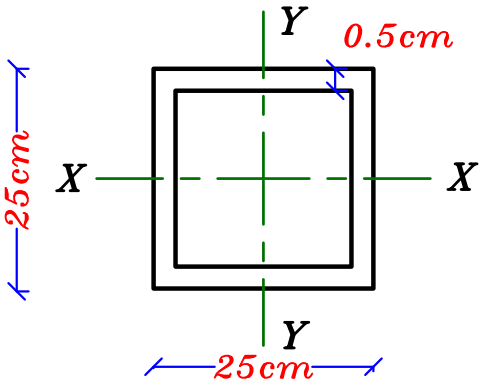


straining actions for shear stresses

Properties of area

$$A = 25 * 25 - 24 * 24 = 49 \ cm^2$$

$$I_x = I_y = \frac{25 * 25^3}{12} - \frac{24 * 24^3}{12} = 4905 \ cm^4$$



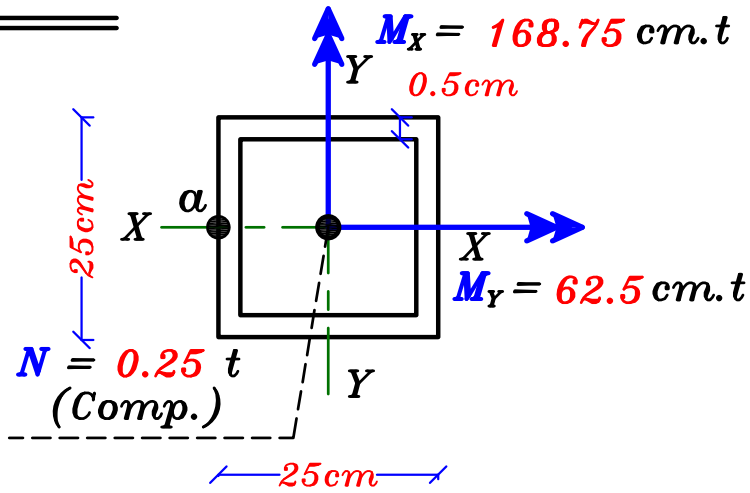
Point (a) at sec A–A

Normal stresses

$$\# \ f = \pm \frac{N}{A} \pm \frac{M_x}{I_x} y \pm \frac{M_y}{I_y} x$$

$$f_a = -\frac{0.25}{49} + \frac{168.75}{4905} * 0 + \frac{62.5}{4905} * 12.5$$

$$= +0.154 \ t\backslash cm^2 \quad \text{شد}$$



Shear stresses

نحسب الـ Shear Stresses نتيجة Q_x & Q_y ثم نحسب الـ Shear Stresses نتيجة M_t ثم نجمعهم معا .

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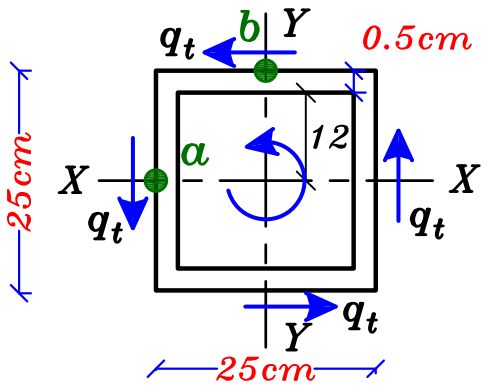
Shear stresses due to Torsion

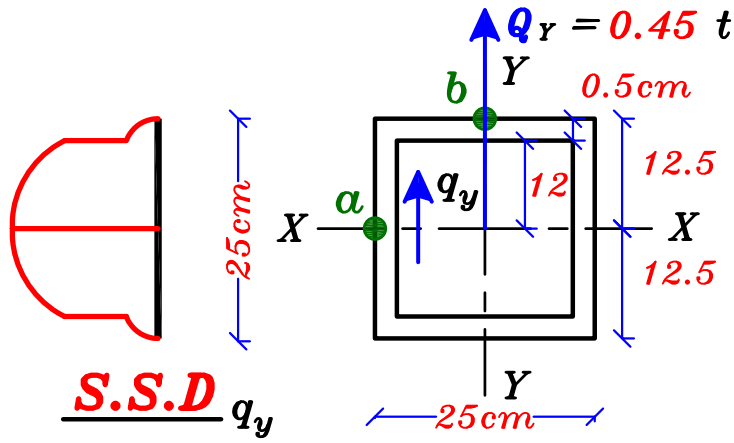


$$q_t = \frac{M_t}{2\,A_{mean}\,t}$$

$$= \frac{112.5}{(2)(24.5*24.5)(0.5)}$$

$$= 0.187\ t\backslash cm^2$$

Shear stresses due to forces



S.S.D q_y

$$\#\ q_y = \frac{Q_y}{I} \times \frac{S}{b}$$

$$= \frac{0.45}{4905} * \frac{2*(12*0.5)(6)+(25*0.5)(12.25)}{0.5+0.5}$$

$$= 0.02\ t\backslash cm^2$$

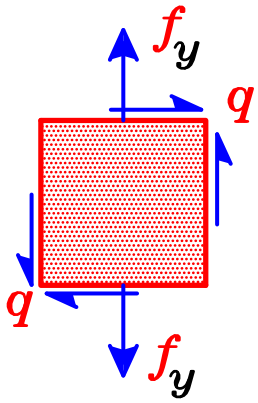
$q_y = 0.02$

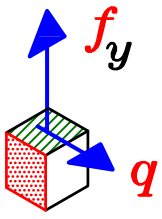
$q_t = 0.187$

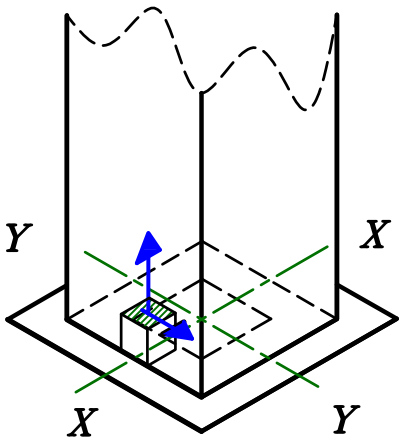
$\Longrightarrow$

$q_{y_{Total}} = 0.166$

For principal stresses







$f_y = +0.154\ t\backslash cm^2$ شد

$f_x = 0$

$q = -0.166\ t\backslash cm^2$

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1)Principle Normal stresses and their planes

$$f_{\substack{max.\\min.}} = \left[\frac{f_x + f_y}{2}\right] \pm \sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$f_{\substack{max.\\min.}} = \left[\frac{0 + 0.154}{2}\right] \pm \sqrt{\left[\frac{0 - 0.154}{2}\right]^2 + (-0.166)^2}$$

$$f_{max.} = 0.26\ t\backslash cm^2$$

$$f_{min.} = -0.106\ t\backslash cm^2$$

$$\boxed{q = 0}$$

$$\tan 2\theta = \frac{-2q}{f_x - f_y} = \frac{-2(-0.166)}{0 - 0.154}$$

$$2\theta = -65^\circ \qquad \boxed{\theta = -32.55^\circ}$$

2)Principle Shear stresses and their planes

$$q_{\substack{max.\\min.}} = \pm \sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$q_{\substack{max.\\min.}} = \pm \sqrt{\left[\frac{0 - 0.154}{2}\right]^2 + (-0.166)^2}$$

$$q_{max.} = 0.183\ t\backslash cm^2 \qquad q_{min.} = -0.183\ t\backslash cm^2$$

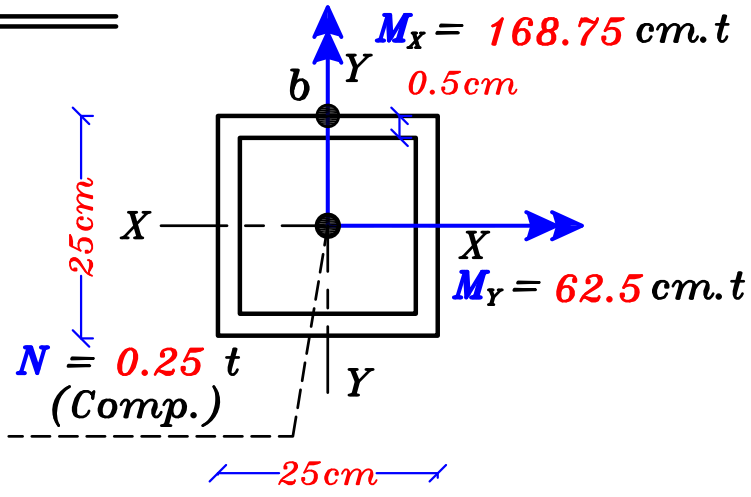
Point (b) at sec A–A

Normal stresses

$$\# f = \pm \frac{N}{A} \pm \frac{M_x}{I_x} y \pm \frac{M_y}{I_y} x$$

$$f_b = -\frac{0.25}{49} - \frac{168.75}{4905} * 12.5 + \frac{62.5}{4905} * 0$$

$$= -0.435\ t\backslash cm^2 \quad \text{ضغط}$$



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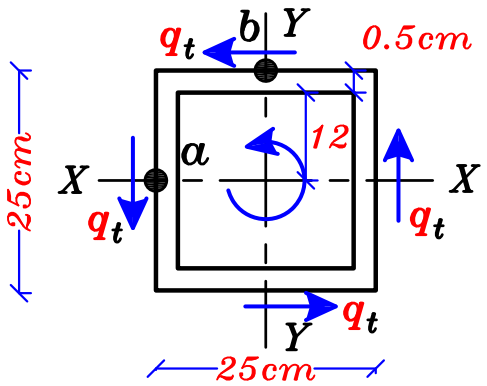
Shear stresses

نحسب الـ Shear Stresses نتيجة Q_x & Q_y ثم نحسب الـ Shear Stresses نتيجة M_t ثم نجمعهم معا .

Shear stresses due to Torsion

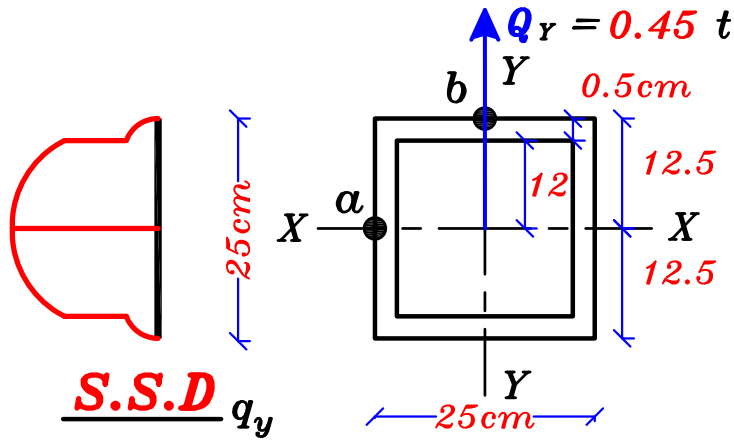


$$q_t = \frac{M_t}{2\,A_{mean}\,t}$$

$$= \frac{112.5}{(2)(24.5*24.5)(0.5)}$$

$$= 0.187\ t\backslash cm^2 \leftarrow$$

Shear stresses due to forces

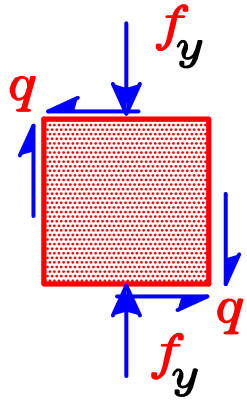


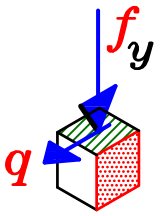
S.S.D q_y

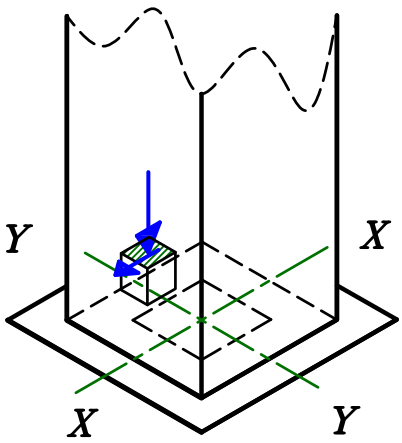
$$\#\ q_y = \frac{Q_y}{I} x \frac{S}{b} = 0$$

$q_{x_{Total}} = 0.187$

For principal stresses







$f_y = -0.435\ t\backslash cm^2$ ضغط

$f_x = 0$

$q = 0.187\ t\backslash cm^2$

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1)Principle Normal stresses and their planes

$$f_{\substack{max.\\min.}} = \left[\frac{f_x + f_y}{2}\right] \pm \sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$f_{\substack{max.\\min.}} = \left[\frac{0 - 0.435}{2}\right] \pm \sqrt{\left[\frac{0 + 0.435}{2}\right]^2 + (0.187)^2}$$

$$f_{max.} = 0.07\ t\backslash cm^2$$

$$f_{min.} = -0.504\ t\backslash cm^2$$

$$\boxed{q = 0}$$

$$\tan 2\theta = \frac{-2q}{f_x - f_y} = \frac{-2(0.187)}{0 + 0.435}$$

$$2\theta = -40^\circ \qquad \boxed{\theta = -20^\circ}$$

2)Principle Shear stresses and their planes

$$q_{\substack{max.\\min.}} = \pm \sqrt{\left[\frac{f_x - f_y}{2}\right]^2 + q^2}$$

$$q_{\substack{max.\\min.}} = \pm \sqrt{\left[\frac{0 + 0.435}{2}\right]^2 + (0.187)^2}$$

$$q_{max.} = 0.286\ t\backslash cm^2 \qquad q_{min.} = -0.286\ t\backslash cm^2$$

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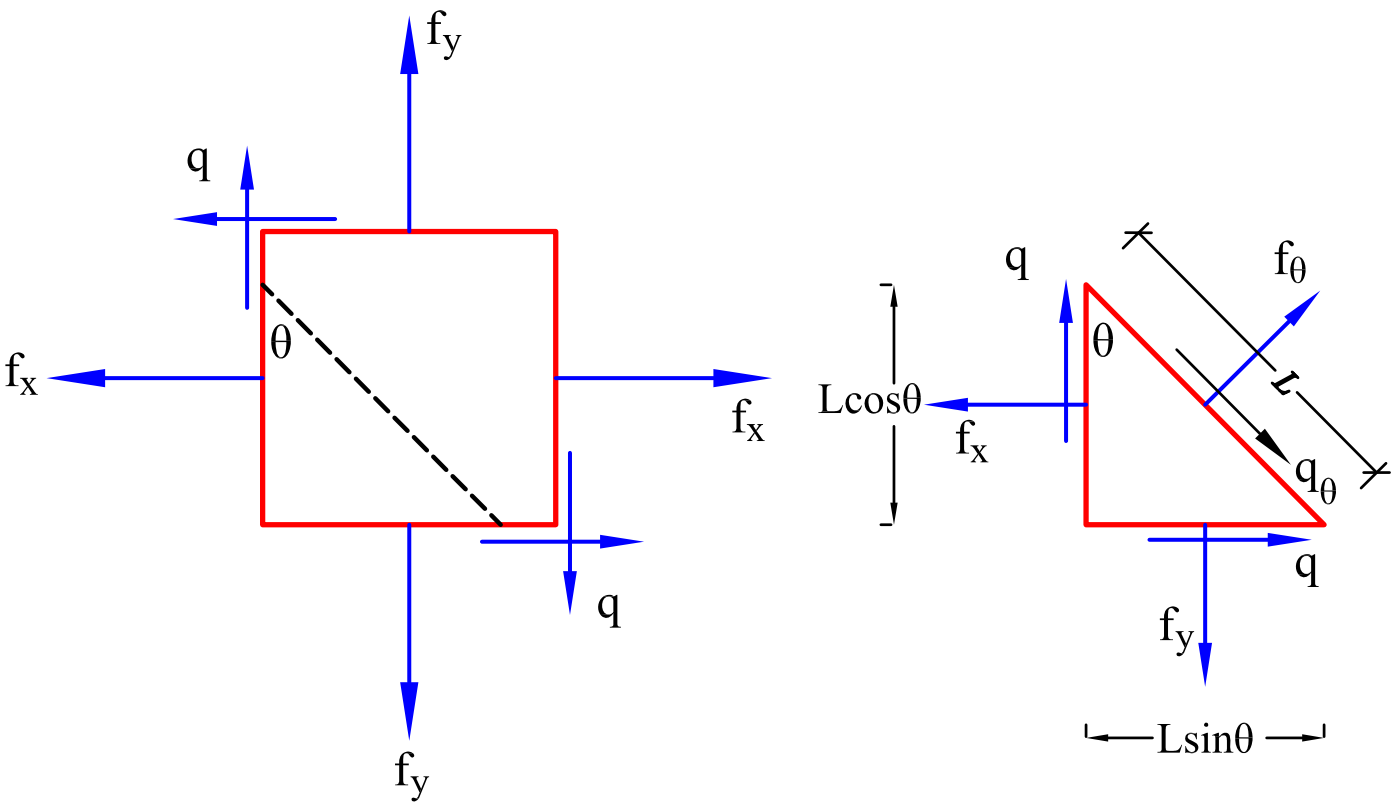
Proof of principle stresses and their planes



$\Sigma X = 0$

$$Fx\,(L\cos\theta)(1) = q\,(L\sin\theta)(1) + F_\theta\,(L)(1)\cos\theta + q_\theta\,(L)(1)\sin\theta$$

$$F_\theta\cos\theta + q_\theta\sin\theta = Fx\cos\theta - q\sin\theta \quad \text{---------- 1)}$$

$\Sigma Y = 0$

$$Fy\,(L\sin\theta)(1) + q_\theta\,(L)(1)\cos\theta = q\,(L\cos\theta)(1) + F_\theta\,(L)(1)\sin\theta$$

$$F_\theta\sin\theta - q_\theta\cos\theta = F\,y\sin\theta - q\cos\theta \quad \text{---------- (2)}$$

Solve (1) & (2)

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نصل للمعادلة (٣) عن طريق ضرب المعادلة (١) فى Cos θ

والمعادلة (٢) فى Sin θ والجمع بين المعادلتين

$$F_\theta \ (\cos^2\theta + \sin^2\theta) = Fx \cos^2\theta + F\,y \sin^2\theta - 2q \sin\theta \cos\theta$$

$$F_\theta = Fx \cos^2\theta + F\,y \sin^2\theta - q \sin 2\theta \ \text{---------} \ (3)$$

$$\sin^2 \theta = (\tfrac{1}{2} - \tfrac{1}{2} \cos 2\theta)$$

$$\cos^2 \theta = (\tfrac{1}{2} + \tfrac{1}{2} \cos 2\theta)$$

$$F_\theta = Fx (\tfrac{1}{2} + \tfrac{1}{2} \cos 2\theta) + F\,y (\tfrac{1}{2} - \tfrac{1}{2} \cos 2\theta) - q \sin 2\theta$$

$$\boxed{F_\theta = \left(\frac{f_x + f_y}{2} \right) + \left(\frac{f_x - f_y}{2} \right) \cos 2\theta - q \sin 2\theta} \quad (4)$$

نصل للمعادلة (٥) عن طريق ضرب المعادلة (١) فى Sin θ

والمعادلة (٢) فى -Cos θ والجمع بين المعادلتين

$$q_\theta \ (\cos^2\theta + \sin^2\theta) = Fx \ Sin\theta \ Cos\theta - Fy \ Sin\theta \ Cos\theta - q \ (\sin^2 \theta - \cos^2 \theta)$$

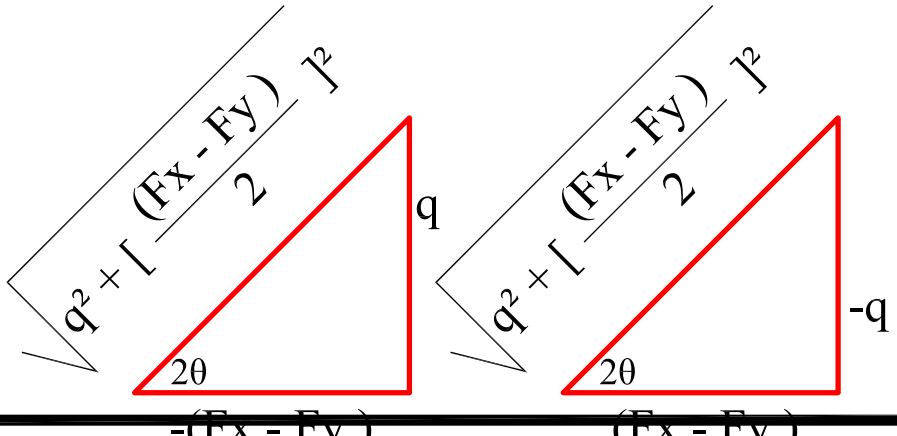
$$\boxed{q_\theta = \left(\frac{f_x - f_y}{2} \right) \sin 2\theta + q \cos 2\theta \ \text{-------} (5)}$$

لايجاد Fmax.&Fmin. نفاضل معادلة F_θ و نساويها بال Zero

$$dF_\theta / d\theta = 0$$

$$\tan 2\theta = \frac{-q}{(Fx - Fy) / 2}$$

equation no. (6)



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$$\sin 2\theta = \frac{-q}{\sqrt{q^2 + [\frac{(Fx - Fy)}{2}]^2}}$$

$$\cos 2\theta = \frac{\frac{(Fx - Fy)}{2}}{\sqrt{q^2 + [\frac{(Fx - Fy)}{2}]^2}}$$

$$\sin 2\theta = \frac{q}{\sqrt{q^2 + [\frac{(Fx - Fy)}{2}]^2}}$$

$$\cos 2\theta = \frac{\frac{-(Fx - Fy)}{2}}{\sqrt{q^2 + [\frac{(Fx - Fy)}{2}]^2}}$$

هذه المعادلات تعطى الزاوية التى يميل بها المستويات المعرضة لـ Fmax.&Fmin. عليه المستويات المعرضة لـ Fx &Fy

و لايجاد قيمة Fmax.&Fmin. نعوض عن قيمتى cos2θ,sin2θ فى المعادلة (٤)

$$f_{max.} = (\frac{f_x + f_y}{2}) + \sqrt{(\frac{f_x - f_y}{2})^2 + q^2} \quad --(7)$$

$$f_{min.} = (\frac{f_x + f_y}{2}) - \sqrt{(\frac{f_x - f_y}{2})^2 + q^2} \quad --(8)$$

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$$dq_\theta / d\theta = 0$$

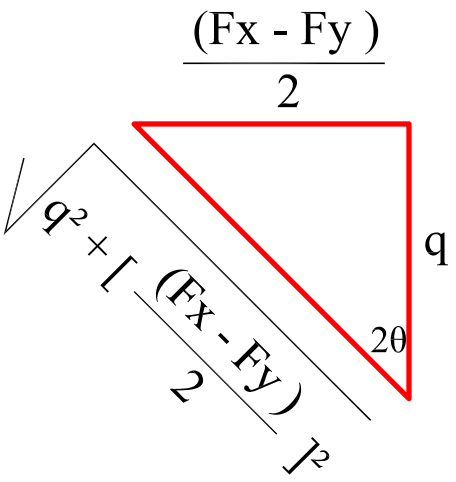
$$\tan 2\theta = \frac{(Fx - Fy)/2}{q} \quad \text{---- (9)}$$

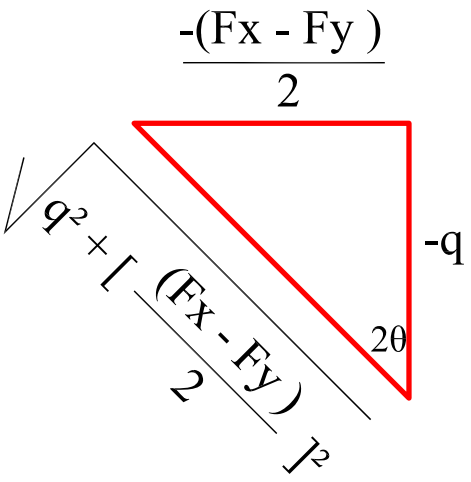
$$\cot 2\theta = \frac{q}{(Fx - Fy)/2} \quad \text{---- (10)}$$

Comparing equation (6)
with equation (10)

$$2\theta_q = 2\theta_F + 90° \quad \text{--- (11)}$$

$$\theta_q - \theta_F = 45°$$





و لايجاد قيمة Fmax.&Fmin. نعوض عن قيمتى cos2θ,sin2θ فى المعادلة (٥)

$$q_{max.} = +\sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + q^2}$$

$$q_{min.} = -\sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + q^2}$$

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